1: Understanding Real Numbers and Basic Arithmetic Properties

Real numbers are real in the sense of representing physical measurements we can perceive in the real world. Believe it or not, there is a class of numbers called imaginary numbers used in some branches of mathematics, but we're going to stick with real numbers here. Within the real numbers, there are rational numbers, which represent fractions or quotients, and irrational numbers, which are any real numbers that aren't rational, such as π or $\sqrt{2}$.

All rational numbers can be written as either terminating or repeating decimals, for example $2/5 = 0.4$ and $1/3 = 0.333 \ldots$. All irrational numbers can only be written as nonterminating and nonrepeating decimals, for example $\pi = 3.141592 \ldots$ and $\sqrt{2} = 1.414213 \ldots$. Within the rational numbers, there are the integers, which can be further divided into positive and negative integers and 0. The positive integers are sometimes called the natural numbers and if we also include 0, the whole numbers.

Any time we're working with numbers, it can be helpful to consider which subset of numbers we're working with, for example, just integers, or just rational numbers, or all real numbers including irrational numbers. Next, let's run through the basic arithmetic properties that will need to become second nature the further you go on your mathematical journey.

- First, it doesn't matter which order we add two numbers together: $a + b = b + a$. For example, $3 + 4$ is the same as $4 + 3$.
- It also doesn't matter which order we add three numbers together: $(a + b) + c = a + (b + c)$. For example, $(3 + 4) + 2$ is the same as $3 + (4 + 2)$.
- If we add 0 to a number, the number doesn't change: $a + 0 = a$.
- If we add a and $-a$, which is the negative of a, we get 0: $a + (-a) = 0$.
- Next, it doesn't matter which order we multiply two numbers together: $ab = ba$. For example, 3×4 is the same as 4×3.
- It also doesn't matter which order we multiply three numbers together: $(ab)c = a(bc)$. For example, $(3 \times 4) \times 2$ is the same as $3 \times (4 \times 2)$.
- If we multiply a number by 1, it doesn't change: $a \times 1 = a$.
- If we multiply a by $1/a$, which is the reciprocal of a, we get 1: $a \times (1/a) = 1$.
- Next, if we multiply a by the sum of b and c, we get the sum of ab and ac: $a(b + c) = ab + ac$. For example, $3(4 + 2) = 3 \times 6 = 18$ or $3(4 + 2) = (3 \times 4) + (3 \times 2) = 12 + 6 = 18$.
- Building from this last property, if we multiply the sum of a and b by the sum of c and d, we get four partial products: $(a + b)(c + d) = ac + ad + bc + bd$. This is referred to as FOIL, standing for "first, outer, inner, last." For example, $(3 + 4) \times (2 + 1) = 7 \times 3 = 21$. Or we can evaluate this using FOIL: $(3 + 4) \times (2 + 1) = (3 \times 2) + (3 \times 1) + (4 \times 2) + (4 \times 1) = 6 + 3 + 8 + 4 = 21$.
- Finally, to solve equations we can add the same number to both sides of an equality or multiply both sides of an equality by the same number: if $a = b$ then $a + c = b + c$ and $ac = bc$. For example, $2x + 3 = 9 \Rightarrow 2x + 3 - 3 = 9 - 3 \Rightarrow 2x = 6 \Rightarrow (1/2)2x = (1/2)6 \Rightarrow x = 3$.