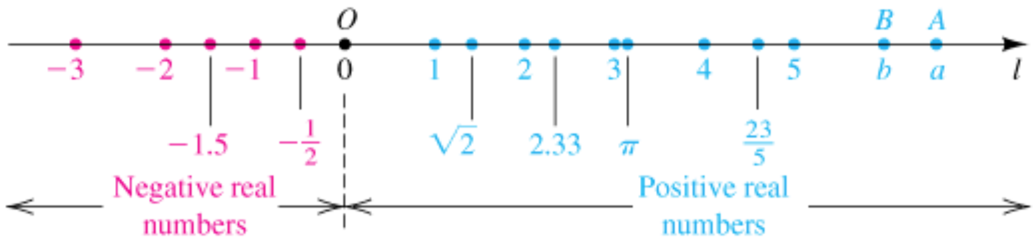
2: Representing Numbers as Points on a Number Line

Thinking of numbers as points along a number line can help us understand arithmetic properties that involve zero, negative numbers, inequalities, and distances. Here's the real number line:



- We go from negative numbers on the left to positive numbers on the right, with 0 in between.
- We can place points to represent integers, such as -3 and 3, rational numbers, such as $-1/2$ and $1/3$, and irrational numbers, such as $\sqrt{2}$ and π.
- The points labeled A and B have associated numbers a and b that represent coordinates.

Sign rules

- Consider multiplying two numbers, a x b. If a and b have the same sign, then the product ab is positive. If a and b have opposite signs, then ab is negative.
- Similarly, if we divide two numbers, a ÷ b. If a and b have the same sign, then the quotient a/b is positive. If a and b have opposite signs, then a/b is negative.

Properties of negatives

- If we multiply a number a by -1 we get $-a$: $(-1)a = -a$. For example, $(-1)(3) = -3$.
- If we take the negative of a negative number, we get the original number again: $-(-a) = a$. For example, $-(-3) = 3$.
- If we multiply $-a$ by b or a by $-b$ we get the same answer: $(-a)b = a(-b) = -(ab)$. For example, $(-3)4 = 3(-4) = -(3x4) = -12$.
- But if we multiply $-a$ by $-b$ we get $(-a)(-b) = ab$. For example, $(-3)(-4) = 12$.

Properties of zero

- If we multiply a number a by 0 we get 0: $a \times 0 = 0$. For example, $3 \times 0 = 0$.
- If $ab = 0$ then $a = 0$ or $b = 0$ (or both $a = b = 0$). This is called the *zero-factor theorem*, and it comes up a lot when solving equations.

Connection between subtraction and addition

- One way to subtract b from a is to add $-b$ to a: $a - b = a + (-b)$. For example, $3 - 4 = 3 + (-4) = -1$ and $3 - (-4) = 3 + 4 = 7$.

Inequalities

- First, strict inequalities: $a > b$ if $a - b$ is positive and $a < b$ if $a - b$ is negative. For example, $3 < 4$ because $3 - 4 = -1$ and $3 > -4$ because $3 - (-4) = 7$.
- If we have two numbers a and b, either $a > b$, $a < b$, or $a = b$.

- Next, nonstrict inequalities: $a \geq b$ if $a > b$ or $a = b$ and $a \leq b$ if $a < b$ or $a = b$.

Absolute values

- $|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$. For example, $|3| = 3$ and $|-4| = 4$. Think of the absolute value as representing the magnitude of a number without regard to its sign.
- This leads to the notion of the distance between two numbers: $d(a, b) = |b - a| = |a - b|$. For example, $d(3,4) = |3 - 4| = |-1| = 1$, $d(3, -4) = |3 - (-4)| = |7| = 7$.