

3: Reducing Fractions to Lowest Terms

Properties of quotients or fractions provide a rationale for simplifying fractions, otherwise known as “reducing them to lowest terms.”

- The first property is that two fractions, a/b and c/d , are equivalent if $ad = bc$. For example, $3/4 = 6/8$ because $3(8) = 4(6) = 24$.
- Next, if we divide $-a$ by b or a by $-b$ we get the same answer: $(-a)/b = a/(-b) = -(a/b)$. For example, $(-3)/4 = 3/(-4) = -(3/4) = -0.75$.
- Finally, if we multiply the numerator and denominator of a fraction, a/b , by the same number, c , we get an equivalent fraction: $a/b = ac/bc$. For example, $3/4 = 3(6)/4(6) = 18/24$.

This last property means that for any fraction, a/b , there are an infinite number of equivalent fractions. However, only one of these is said to be in “lowest terms.” a/b is in lowest terms if a and b have no common factor greater than 1. For example, $3/4$ is in lowest terms because the only common factor for 3 and 4 is 1.

This begs the question; how do we reduce a fraction to lowest terms? We can apply one of the fraction properties from earlier. We can reduce a fraction to lowest terms by repeatedly dividing by common factors: $a/b = (a/d)/(b/d)$. For example, $18/24 = (18/3)/(24/3) = 6/8 = (6/2)/(8/2) = 3/4$. Alternatively, divide the numerator and denominator by 6 to get $18/24 = 18^3/24^4 = 3/4$.

Why reduce a fraction to lowest terms?

In general, it is easier to work with small numbers than large numbers. For example, $3/4$ is easier to work with than $18/24$. The idea of “reducing to lowest terms” comes up again when we consider rational expressions in Video 12.

However, sometimes we don’t want to reduce a fraction to lowest terms. For example, if we want to express $3/4$ as a terminating decimal: $3/4 = 3(25)/4(25) = 75/100 = 0.75$.

More examples

- $\frac{10}{15} = \frac{10/5}{15/5} = \frac{2}{3}$
- $\frac{9}{36} = \frac{9/3}{36/3} = \frac{3/3}{12/3} = \frac{1}{4}$
- Quicker: $\frac{9}{36} = \frac{9/9}{36/9} = \frac{1}{4}$
- $\frac{16}{12} = \frac{16/4}{12/4} = \frac{4}{3} = \frac{3+1}{3} = 1 + \frac{1}{3} = 1\frac{1}{3}$
- $\frac{18}{30} = \frac{18/6}{30/6} = \frac{3}{5}$
- $\frac{77}{42} = \frac{77/7}{42/7} = \frac{11}{6} = 1\frac{5}{6}$