

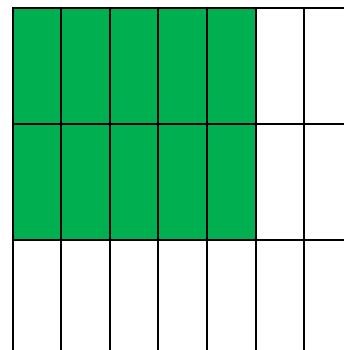
4: Multiplying and Dividing Fractions

Multiplying fractions

To multiply two fractions, simply multiply the numerators and multiply the denominators: $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$. For example, $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$.

Why this makes sense

Read $\frac{2}{3} \times \frac{5}{7}$ as two-thirds of five-sevenths. Take a unit square, mark off 7 columns and block off 5 of them to represent five-sevenths. Then mark off 3 rows and shade the pieces inside 2 rows within the 5 blocked off columns to represent two-thirds of five-sevenths. There are $2 \times 5 = 10$ shaded pieces, which is the numerator of the answer. There are $3 \times 7 = 21$ total pieces, which is the denominator of the answer. So, $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$.



Another example

$\frac{2}{3} \times \frac{5}{8}$. Multiplying the numerators and denominators we get $\frac{2 \times 5}{3 \times 8} = \frac{10}{24}$. We would typically then reduce this to lowest terms using the common factor 2: $\frac{10}{24} = \frac{5}{12}$. Alternatively, we can cancel this common factor before multiplying: $\frac{2^1}{3} \times \frac{5}{8^4} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}$. This technique of canceling common factors is a useful one to remember as it can save a lot of time with more complicated calculations.

Dividing fractions

Consider the connection between dividing whole numbers and multiplying fractions. One way to divide a number, a , by another number, b , is to multiply a by the reciprocal of b . Algebraically, $a \div b = a \times \frac{1}{b}$. For example, $3 \div 4 = 3 \times \frac{1}{4} = \frac{3}{4}$ or 0.75.

We can do something similar when dividing fractions. To divide a fraction, $\frac{a}{b}$, by another fraction, $\frac{c}{d}$, we multiply $\frac{a}{b}$ by the reciprocal of $\frac{c}{d}$. Algebraically, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$. For example, $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}$.

More examples

- $\frac{2}{3} \div \frac{5}{8} = \frac{2}{3} \times \frac{8}{5} = \frac{16}{15} = 1 \frac{1}{15}$
- $\frac{5}{8} \div \frac{2}{3} = \frac{5}{8} \times \frac{3}{2} = \frac{15}{16}$
- $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4}{3} = \frac{20}{24} = \frac{5}{6}$
- Quicker: $\frac{5}{8} \div \frac{3}{4} = \frac{5}{8} \times \frac{4^1}{3} = \frac{5}{6}$
- $\frac{4}{9} \div \frac{5}{6} = \frac{4}{9} \times \frac{6^2}{5} = \frac{8}{15}$
- $\frac{5}{9} \div \frac{4}{15} = \frac{5}{9} \times \frac{15^5}{4} = \frac{25}{12} = 2 \frac{1}{12}$