

5: Adding and Subtracting Fractions

Adding fractions with the same denominator

To add two fractions with the same denominator we simply add the numerators: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$. For example, $\frac{3}{5} + \frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$.



Why this makes sense

Think of $\frac{3}{5}$ as a fraction strip with $\frac{3}{5}$ shaded blue: the blue section is 3 parts out of the whole fraction strip, which is 5 parts. Then think of $\frac{1}{5}$ as another fraction strip with $\frac{1}{5}$ shaded green: the green section is 1 part out of the whole 5 parts. The shaded parts are all the same size, so if we add them together, we get 4 parts out of the whole 5 parts. So, $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$.



Subtracting fractions with the same denominator

Subtracting two fractions with the same denominator is similar; we simply subtract the numerators: $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$. For example, $\frac{3}{5} - \frac{1}{5} = \frac{3-1}{5} = \frac{2}{5}$.

Adding or subtracting fractions with different denominators

For example, how can we add $\frac{3}{5}$ and $\frac{1}{3}$? We can't simply add the numerators because the parts are now different sizes. Instead, to add two fractions with different denominators, $\frac{a}{b}$ and $\frac{c}{d}$, we need to find equivalent fractions that have a common denominator. An easy way to do that is to multiply the first fraction, $\frac{a}{b}$, by $\frac{d}{d}$, where d is the denominator of the second fraction. Then, multiply the second fraction, $\frac{c}{d}$, by $\frac{b}{b}$, where b is the denominator of the first fraction. Now we have new common denominators equal to bd and we simply add the new numerators, which are ad and bc : $\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \left(\frac{d}{d}\right) + \left(\frac{b}{b}\right) \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$. For example, $\frac{3}{5} + \frac{1}{3} = \frac{3}{5} \left(\frac{3}{3}\right) + \left(\frac{5}{5}\right) \frac{1}{3} = \frac{9}{15} + \frac{5}{15} = \frac{14}{15}$.



Why this makes sense

Multiplying $\frac{3}{5}$ by $\frac{3}{3}$ creates an equivalent fraction of $\frac{9}{15}$ shown by 9 blue parts out of 15. Similarly, multiplying $\frac{1}{3}$ by $\frac{5}{5}$ creates an equivalent fraction of $\frac{5}{15}$ shown by 5 green parts out of 15. The shaded parts are all the same size now, so if we add them together, we get 14 parts out of the whole 15 parts. So, $\frac{3}{5} + \frac{1}{3} = \frac{9}{15} + \frac{5}{15} = \frac{14}{15}$.



More examples

- $\frac{1}{3} + \frac{3}{8} = \frac{1}{3} \left(\frac{8}{8}\right) + \left(\frac{3}{3}\right) \frac{3}{8} = \frac{8}{24} + \frac{9}{24} = \frac{17}{24}$
- $\frac{3}{5} - \frac{2}{7} = \frac{3}{5} \left(\frac{7}{7}\right) - \left(\frac{5}{5}\right) \frac{2}{7} = \frac{21}{35} - \frac{10}{35} = \frac{11}{35}$

- $\frac{5}{8} + \frac{1}{6} = \frac{5}{8} \binom{6}{6} + \binom{8}{8} \frac{1}{6} = \frac{38}{48} = \frac{19}{24}.$
- Quicker: $\frac{5}{8} + \frac{1}{6} = \frac{5}{8} \binom{3}{3} + \binom{4}{4} \frac{1}{6} = \frac{19}{24}.$
- $\frac{1}{4} + \frac{4}{7} = \frac{1}{4} \binom{7}{7} + \binom{4}{4} \frac{4}{7} = \frac{7}{28} + \frac{16}{28} = \frac{23}{28}$
- $\frac{5}{9} - \frac{1}{3} = \frac{5}{9} - \binom{3}{3} \frac{1}{3} = \frac{5}{9} - \frac{3}{9} = \frac{2}{9}$