6: Understanding Integer Exponents

Positive exponents

The expression "a with exponent n" means "a times a times a, and so on, n times:" $a^n = \underbrace{a \times a \times \ldots \times a}_{n \text{ factors}}$. Here a is the *base* (any real number) and n is the *exponent*. We can also say "a raised to the power n" or simply "a to the n." For example, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.

More examples

- $\left(\frac{1}{2}\right)^5 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$
- $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$
- $\pi^5 = \pi \times \pi \times \pi \times \pi \times \pi$

Special cases

- If $n = 1$, we have a special case with just one factor: $a^1 = a$. For example, $2^1 = 2$.
- If $n = 0$: $a^0 = 1$ for all real numbers a except 0 (although in many branches of math, 0^0 is also defined to be 1). For example, $2^0 = 1$.

Negative exponents

- If $n = -1$, a^{-1} is defined as the reciprocal of a: $a^{-1} = \frac{1}{a}$, for $a \neq 0$. For example, $2^{-1} = 1/2$.
- More generally, a^{-n} is defined as the reciprocal of a^n: $a^{-\mathrm{n}} = \frac{1}{a^n}$, for $a \neq 0$. For example, $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$.
- Equivalently, $a^{-\mathrm{n}} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$. For example, $2^{-5} = \left(\frac{1}{2}\right)^5$.
- Similarly, $\left(\frac{1}{a}\right)^{-\mathrm{n}} = a^n$. For example, $\left(\frac{1}{2}\right)^{-5} = 2^5$.

More examples

- $(-2)^{-5} = \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) \times \left(-\frac{1}{2}\right) = -\frac{1}{32}$
- $\pi^{-5} = \frac{1}{\pi} \times \frac{1}{\pi} \times \frac{1}{\pi} \times \frac{1}{\pi} \times \frac{1}{\pi} = \frac{1}{\pi^5}$

More complex expressions

With more complex expressions it can be simpler to use a "dot" to represent multiplication:

- $ba^{\mathrm{n}} = b \cdot a^{\mathrm{n}} \neq (ba)^n$. For example, $3 \cdot 2^5 = 3 \cdot 32 = 96$. However, $(3 \cdot 2)^5 = 6^5 = 7776$.
- $ba^{-\mathrm{n}} = b \cdot a^{-\mathrm{n}} = b \cdot \frac{1}{a^n} = \frac{b}{a^n}$. For example, $3 \cdot 2^{-5} = 3 \cdot \frac{1}{2^5} = \frac{3}{2^5} = \frac{3}{32}$.
- $\frac{b}{a^{-\mathrm{n}}} = b \cdot \frac{1}{a^{-n}} = b \cdot \left(\frac{1}{a}\right)^{-n} = ba^n$. For example, $\frac{3}{2^{-5}} = 3 \cdot \frac{1}{2^{-5}} = 3 \cdot \left(\frac{1}{2}\right)^{-5} = 3 \cdot 2^5 = 3 \cdot 32$.

More examples

- $4(-2)^{-3} = \frac{4}{(-2)^3} = \frac{4}{(-2)\times(-2)\times(-2)} = \frac{4}{-8} = -\frac{1}{2}$
- $\frac{-3}{(-4)^{-2}} = -3(-4)^2 = -3 \cdot 16 = -48$