10: Multiplying Polynomials

Multiplying polynomials that each have two terms

Recall how to multiply the sum of two integers, a and b, by the sum of two integers, c and d: $(a+b)(c+d) = ac + ad + bc + bd$. We refer to these four partial products as FOIL, standing for "first, outer, inner, last." For example, $(3+4) \times (2+1) = (3 \times 2) + (3 \times 1) + (4 \times 2) + (4 \times 1) = 6 + 3 + 8 + 4 = 21$.

Let's apply this idea to multiplying polynomials, $2x^2 + 3$ and $x - 4$:

- $(2x^2+3)(x-4) = (2x^2)(x) + (2x^2)(-4) + (3)(x) + (3)(-4) = 2x^3 - 8x^2 + 3x - 12.$

More examples

- $(x^2 - 2x)(-3x+1) = (x^2)(-3x) + (x^2)(1) + (-2x)(-3x) + (-2x)(1)$
  $= -3x^3 + x^2 + 6x^2 - 2x = -3x^3 + 7x^2 - 2x.$
- $(3x-2)^2 = (3x-2)(3x-2) = (3x)(3x) + (3x)(-2) + (-2)(3x) + (-2)(-2)$
  $= 9x^2 - 6x - 6x + 4 = 9x^2 - 12x + 4$. Here, two of the partial products are combined.
- $(4x+3)(4x-3) = (4x)(4x) + (4x)(-3) + (3)(4x) + (3)(-3)$
  $= 16x^2 - 12x + 12x - 9 = 16x^2 - 9$. Here, two of the partial products cancel out to zero. The term $4x - 3$ is called the *conjugate* of $4x + 4$.

Multiplying polynomials with more than two terms

When there are more than two terms in the polynomials, we must make sure we remember to multiply each term in one polynomial by each term in the other polynomial. For example:

- $(-2x^3 - 3x + 4)(3x^2 - 1)$
  $= (-2x^3)(3x^2) + (-2x^3)(-1) + (-3x)(3x^2) + (-3x)(-1) + (4)(3x^2) + (4)(-1)$
  $= -6x^5 + 2x^3 - 9x^3 + 3x + 12x^2 - 4 = -6x^5 - 7x^3 + 12x^2 + 3x - 4.$

Polynomial multiplication and addition together

- $(-x+3)(4x^2+3x) - 2x(-3x^2+2x+4) = -4x^3 - 3x^2 + 12x^2 + 9x + 6x^3 - 4x^2 - 8x$
  $= 2x^3 + 5x^2 + x.$
- Check by plugging in the value $x = 2$: $1(22) - 4(-4) = 38 \Rightarrow 16 + 20 + 2 = 38 \Rightarrow 38 = 38.$

More examples

- $(-4x^3 + x)(2x^2 - 3) = (-4x^3)(2x^2) + (-4x^3)(-3) + (x)(2x^2) + (x)(-3)$
  $= -8x^5 + 12x^3 + 2x^3 - 3x = -8x^5 + 14x^3 - 3x.$
- $-3x^2(2x^2 - x - 4) + (2x^2 + 3x)(2x^2 - 3x) = -6x^4 + 3x^3 + 12x^2 + 4x^4 - 9x^2$
  $= -2x^4 + 3x^3 + 3x^2$. Note the conjugate in this example.