

## 11: Factoring Polynomials

### Factors of polynomials

- If we multiply two polynomials together, we obtain another polynomial as the result. For example,  $2x^2(x - 4) = 2x^3 - 8x^2$ .
- A *factor* is a polynomial that divides evenly into the polynomial result. So,  $2x^2$  and  $x - 4$  are factors of  $2x^3 - 8x^2$ .
- Going in reverse to find the factors that multiply together to create the polynomial result is called *factoring the polynomial*.

### Factoring a polynomial

We typically start by trying to find a common factor of each term in the polynomial, if there is one. In this case,  $2x^2$  is a common factor of  $2x^3$  and  $-8x^2$ :

- $2x^3 = 2x^2(x)$
- $-8x^2 = 2x^2(-4)$

Therefore,  $2x^3 - 8x^2 = 2x^2(x - 4)$ .

Factoring is useful for finding when the polynomial equals zero, in this case when  $x = 0$  or  $x = 4$ .

Another example:  $3x^3 - 3x^2 + 6x = 3x(x^2 - x + 2)$ .

### Factoring a quadratic polynomial

Degree 2 polynomials are called quadratics. For example, let's try to factor  $x^2 + 5x + 6$  as  $(x + a)(x + b)$ . We need to find constants  $a, b$  such that  $a + b = 5$  and  $ab = 6$ . Two values that work are  $a = 3$  and  $b = 2$ . Check:  $(x + 3)(x + 2) = x^2 + 5x + 6$ .

Another example:  $x^2 + 3x - 10 = (x + 5)(x - 2)$ .

### Factoring a difference of squares

When multiplying  $(4x + 3)$  by its conjugate  $(4x - 3)$  the result is  $16x^2 - 9 = (4x)^2 - 3^2$ . This gives a quick way to factor a difference of squares.

For example,  $9x^2 - 4 = (3x)^2 - 2^2 = (3x + 2)(3x - 2)$ .

Another example:  $4x^2 - 25 = (2x + 5)(2x - 5)$ .

### Factoring using radicals

We've considered only examples with integer coefficients up to now. But sometimes it is also possible to factor using radicals, for example,  $x^2 - 5 = (x + \sqrt{5})(x - \sqrt{5})$ .

### Factoring polynomials with more than one variable

Also, we've considered only examples with one variable up to now. But it is also possible to factor polynomials with more than one variable, for example,  $6x^3y - 15xy^2 = 3xy(2x^2 - 5y)$ .

### More examples

- $2x(x - 4) + 3(x - 4) = (2x + 3)(x - 4)$
- $2x^3(x - 3) + 2x(x - 3)^2 = 2x(x - 3)(x^2 + x - 3)$

- $2x^3 - 2x^2 - 24x = 2x(x^2 - x - 12) = 2x(x + 3)(x - 4).$
- $3x^3y - 12xy^3 = 3xy(x^2 - 4y^2) = 3xy(x + 2y)(x - 2y).$
- $3x^2 + 9x - 12 = 3(x^2 + 3x - 4) = 3(x + 4)(x - 1)$
- $x^3 + 2x^2 - 9x - 18 = (x^3 - 9x) + (2x^2 - 18)$   
 $= x(x^2 - 9) + 2(x^2 - 9) = (x + 2)(x^2 - 9) = (x + 2)(x + 3)(x - 3).$