12: Dividing Polynomials and Simplifying Rational Expressions

Dividing polynomials to obtain a rational expression

If we multiply the two polynomials, $2x^2 + 3$ and $x - 4$, we obtain another polynomial, $2x^3 - 8x^2 + 3x - 12$.

However, when we divide two polynomials, we generally don't obtain another polynomial. Rather we obtain another type of algebraic expression called a *rational expression*.

For example, $(2x^2 + 3) \div (x - 4) = \frac{2x^2+3}{x-4}$. One thing to remember about a rational expression is that it is not defined when the denominator is 0, in this case when $x = 4$.

Simplifying rational expressions

We can't simplify the rational expression $\frac{2x^2+3}{x-4}$. However, sometimes we can apply ideas about factoring polynomials to simplify a rational expression.

For example, $\frac{2x^3+6x^2}{2x^2} = \frac{\cancel{2x^2}(x+3)}{\cancel{2x^2}} = x + 3$.

However, we must remember that this expression is not defined when $2x^2 = 0$, i.e., when $x = 0$.

More examples

- $\frac{x^2-4}{x^2+3x-10} = \frac{(x+2)\cancel{(x-2)}}{(x+5)\cancel{(x-2)}} = \frac{x+2}{x+5}, \{\forall\ x \neq -5, 2\}$ [Note: this notation means "for all x not equal to -5 or 2," i.e., x is restricted to the set of real numbers except -5 and 2.]
- $\frac{x^2+x-12}{x^2-6x+9} = \frac{(x+4)\cancel{(x-3)}}{(x-3)\cancel{(x-3)}} = \frac{x+4}{x-3}, \{\forall\ x \neq 3\}$.
- $\frac{(x^2+4x+4)(2x+2)}{(x^2-1)(x+2)} = \frac{(x+2)^{\cancel{2}}2\cancel{(x+1)}}{\cancel{(x+1)}(x-1)\cancel{(x+2)}} = \frac{2(x+2)}{x-1}, \{\forall\ x \neq -2, -1, 1\}$.
- $\frac{3x^3+3x^2-18x}{6x^2-12x} = \frac{3x(x^2+x-6)}{6x(x-2)} = \frac{3\cancel{x}(x+3)\cancel{(x-2)}}{\cancel{6}^2\cancel{x}\cancel{(x-2)}} = \frac{x+3}{2}, \{\forall\ x \neq 0, 2\}$.
- $\frac{(2x-6)(3x^2+2x)}{(3x+2)(x^2-9)} = \frac{2\cancel{(x-3)}x\cancel{(3x+2)}}{\cancel{(3x+2)}(x+3)\cancel{(x-3)}} = \frac{2x}{x+3}, \left\{\forall\ x \neq -\frac{2}{3}, -3, 3\right\}$.