18: Solving Quadratic Equations Using the Quadratic Formula

The quadratic formula

We can always solve quadratic equations using the quadratic formula when factoring the quadratic equation either doesn't work or isn't particularly easy.

The quadratic formula, $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, gives solutions to the equation $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

- If the discriminant $b^2 - 4ac > 0$, then there are two real solutions.
- If $b^2 - 4ac = 0$, there is one real solution.
- If $b^2 - 4ac < 0$, there are no real solutions.

Examples

Find any roots of $x^2 + x - 6 = 0$:

- $x = \frac{-1 \pm \sqrt{1^2-4(1)(-6)}}{2(1)} = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} = -3, 2$. Two real solutions.
- Check: $(-3)^2 + (-3) - 6 = 9 - 3 - 6 = 0$, $2^2 + 2 - 6 = 4 + 2 - 6 = 0$.

Find any roots of $4x^2 - 12x + 9 = 0$:

- $x = \frac{12 \pm \sqrt{(-12)^2-4(4)(9)}}{2(4)} = \frac{12 \pm \sqrt{0}}{8} = \frac{3}{2}$. One real solution.
- Check: $4\left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right) + 9 = 9 - 18 + 9 = 0$.

Find any roots of $x^2 + 4x + 5 = 0$:

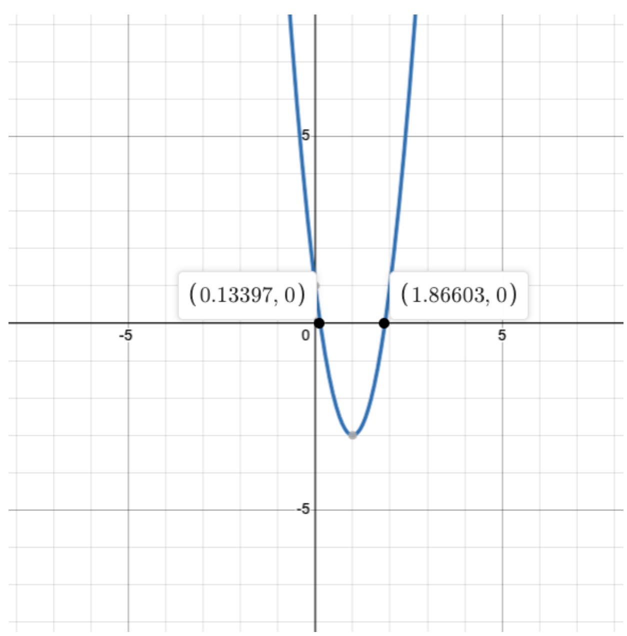
- $x = \frac{-1 \pm \sqrt{1^2-4(1)(5)}}{2(1)} = \frac{-1 \pm \sqrt{-19}}{2}$. No real solutions.

When factoring is not possible

Using the quadratic formula is most useful when factoring is not possible. For example:

Find any roots of $4x^2 - 8x + 1 = 0$:

- $x = \frac{8 \pm \sqrt{(-8)^2-4(4)(1)}}{2(4)} = \frac{8 \pm \sqrt{48}}{8} = \frac{8 \pm \sqrt{16(3)}}{8} = \frac{8 \pm 4\sqrt{3}}{8} = 1 \pm \frac{\sqrt{3}}{2} = 1 - \frac{\sqrt{3}}{2}, 1 + \frac{\sqrt{3}}{2} = 0.134, 1.866$.
- Check: $4(0.134)^2 - 8(0.134) + 1 = 0.072 - 1.072 + 1 = 0$, $4(1.866)^2 - 8(1.866) + 1 = 13.928 - 14.928 + 1 = 0$.



When factoring isn't easy

It is also useful when factoring isn't particularly easy. For example:

Find any roots of $24x^2 + 22x - 35 = 0$:

- $x = \frac{-22 \pm \sqrt{22^2 - 4(24)(-35)}}{2(24)} = \frac{-22 \pm \sqrt{3844}}{48} = \frac{-22 \pm 62}{48} = \frac{-11 \pm 31}{24} = -\frac{42}{24}, \frac{20}{24} = -\frac{7}{4}, \frac{5}{6}$.
- So, $24x^2 + 22x - 35 = (4x + 7)(6x - 5)$.
- Check: $24\left(-\frac{7}{4}\right)^2 + 22\left(-\frac{7}{4}\right) - 35 = \frac{147}{2} - \frac{77}{2} - \frac{70}{2} = 0$,
  $24\left(\frac{5}{6}\right)^2 + 22\left(\frac{5}{6}\right) - 35 = \frac{50}{3} + \frac{55}{3} - \frac{105}{3} = 0$.

More examples

Find any roots of $2x^2 + 3x - 1 = 0$:

- $x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)} = \frac{-3 \pm \sqrt{17}}{4} = -1.781, 0.281$. Two solutions.
- Check: $2(-1.781)^2 + 3(-1.781) - 1 = 6.344 - 5.343 - 1 \approx 0$,
  $2(0.281)^2 + 3(0.281) - 1 = 0.158 + 0.843 - 1 \approx 0$.

Find any roots of $10x^2 - 19x + 6 = 0$:

- $x = \frac{19 \pm \sqrt{(-19)^2 - 4(10)(6)}}{2(10)} = \frac{19 \pm \sqrt{121}}{20} = \frac{19 \pm 11}{20} = \frac{2}{5}, \frac{3}{2}$. Two solutions.
- $10x^2 - 19x + 6 = (5x - 2)(2x - 3)$.
- Check: $10\left(\frac{2}{5}\right)^2 - 19\left(\frac{2}{5}\right) + 6 = \frac{8}{5} - \frac{38}{5} + \frac{30}{5} = 0$,
  $10\left(\frac{3}{2}\right)^2 - 19\left(\frac{3}{2}\right) + 6 = \frac{45}{2} - \frac{57}{2} + \frac{12}{2} = 0$.