7: Applying Exponent Properties

Multiplying and dividing numbers with exponents

- When multiplying numbers with exponents, add the exponents if the base number a is the same: $a^{\mathrm{m}} \times a^{\mathrm{n}} = a^{m+n}$. For example, $2^3 \times 2^2 = 2^5$. This makes sense because if we write it out in full, we get: $(2 \times 2 \times 2) \times (2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2$.
- When dividing numbers with exponents, subtract the exponents if the base number a is the same: $a^{\mathrm{m}} \div a^{\mathrm{n}} = \frac{a^{\mathrm{m}}}{a^{\mathrm{n}}} = a^{m-n}$. For example, $2^5 \div 2^3 = 2^2$. This makes sense because if we write it out in full, we get: $(2 \times 2 \times 2 \times 2 \times 2) \div (2 \times 2 \times 2) = 2 \times 2$.

Applying an exponent to a number with an exponent

- When applying an exponent to a number that itself has an exponent, multiply the exponents: $(a^m)^n = a^{mn}$. For example, $(2^3)^2 = 2^6$. This makes sense because if we write it out in full, we get: $(2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

More examples

- $(-2)^2(-2)^3 = (-2)^{2+3} = (-2)^5 = -32$
- $2^2 2^{-3} = 2^{2-3} = 2^{-1} = \frac{1}{2}$
- $\frac{(-2)^2}{(-2)^3} = (-2)^{2-3} = (-2)^{-1} = -\frac{1}{2}$
- $\frac{2^{-2}}{2^3} = 2^{-2-3} = 2^{-5} = \frac{1}{32}$
- $((-2)^2)^3 = (-2)^{2(3)} = (-2)^6 = 64$
- $(2^{-2})^3 = 2^{-2(3)} = 2^{-6} = \frac{1}{64}$

More complicated expressions with variables

- $3x^2 5x^4 = 3 \times x^2 \times 5 \times x^4 = (3 \times 5) \times (x^2 \times x^4) = 15x^6$
- $4x^5 3x^{-2} = (4 \times 3) \times (x^5 \times x^{-2}) = 12x^3$
- $\frac{4x^3}{2x^5} = \left(\frac{4}{2}\right)\left(\frac{x^3}{x^5}\right) = 2x^{-2} = \frac{2}{x^2}$
- $\frac{xy^5}{x^2y^3} = \left(\frac{x}{x^2}\right)\left(\frac{y^5}{y^3}\right) = x^{-1}y^2 = \frac{y^2}{x}$
- $\frac{x^4y^2}{x^{-3}y^5} = \frac{x^7}{y^3}$

Applying an exponent to a product or a quotient

- When applying an exponent to a product, apply the exponent to each factor separately: $(ab)^n = a^n b^n$. For example, $(4 \times 2)^2 = 8^2 = 64$ or $(4 \times 2)^2 = 4^2 \times 2^2 = 16 \times 4 = 64$. This makes sense because if we write it out in full, we get: $(4 \times 2) \times (4 \times 2) = 4 \times 2 \times 4 \times 2 = (4 \times 4) \times (2 \times 2)$.

- When applying an exponent to a quotient, apply the exponent to the numerator and denominator separately: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$. For example, $\left(\frac{4}{2}\right)^2 = 2^2 = 4$ or $\left(\frac{4}{2}\right)^2 = \frac{4^2}{2^2} = \frac{16}{4} = 4$. This makes sense because if we write it out in full, we get: $\left(\frac{4}{2}\right) \times \left(\frac{4}{2}\right) = 4 \times \left(\frac{1}{2}\right) \times 4 \times \left(\frac{1}{2}\right) = (4 \times 4) \times ((1/2) \times (1/2)) = (4 \times 4)/(2 \times 2)$.

More examples

- $(2xy^{-1})^{-2}(x^{-2}y^2)^3 = 2^{-2}x^{-2}(y^{-1})^{-2}(x^{-2})^3(y^2)^3 = \frac{y^8}{4x^8} = \frac{1}{4}\left(\frac{y}{x}\right)^8$
- $\frac{(2x^3y)^4}{5(xy^2)^2} = \frac{2^4(x^3)^4y^4}{5x^2(y^2)^2} = \frac{16x^{12}y^4}{5x^2y^4} = \frac{16x^{10}}{5} = \frac{16}{5}x^{10}$
- $(xy^2)^3(2x^4y)^{-1} = x^3(y^2)^3 2^{-1}(x^4)^{-1}y^{-1} = 2^{-1}x^{-1}y^5 = \frac{y^5}{2x}$
- $\frac{5(x^2y^{-2})^3}{(-3x^{-1}y)^2} = \frac{5(x^2)^3(y^{-2})^3}{(-3)^2(x^{-1})^2y^2} = \frac{5x^8}{9y^8} = \frac{5}{9}\left(\frac{x}{y}\right)^8$