9: Adding and Subtracting Polynomials

Definition and terminology of polynomials

The algebraic expression, $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$, represents a *polynomial* of *degree* n in the *variable* x. Here n is a non-negative integer.

- The a's, called *coefficients*, are real number constants with leading coefficient $a_n \neq 0$.
- Each product of a coefficient and a power of x is called a *term* with *leading term* $a_n x^n$ and *constant term* a_0.

For example, $2x^3 - 4x^2 + 1$:

- Degree 3.
- Leading term $2x^3$ with leading coefficient 2.
- Coefficient of x^2 is -4.
- Coefficient of x is 0.
- Constant term is 1.

Adding and subtracting polynomials

To add and subtract polynomials, we must compare terms with the same power of the variable. For example, to add $2x^3 - 4x^2 + 1$ and $3x^3 + 5x^2 + 2x - 4$:

- Add $2x^3$ and $3x^3$ to get $(2+3)x^3 = 5x^3$.
- Add $-4x^2$ and $5x^2$ to get $(-4+5)x^2 = x^2$.
- Add $0x$ and $2x$ to get $(0+2)x = 2x$.
- Add 1 and -4 to get $1 + (-4) = -3$.
- The final answer is therefore $5x^3 + x^2 + 2x - 3$.

Similarly, $(2x^3 - 4x^2 + 1) - (3x^3 + 5x^2 + 2x - 4) = -x^3 - 9x^2 - 2x + 5$.

Multiplying a polynomial by a constant

Think of multiplying a polynomial by a constant as repeated addition or subtraction. For example:

- $2(2x^3 - 4x^2 + 1) = (2x^3 - 4x^2 + 1) + (2x^3 - 4x^2 + 1) = 4x^3 - 8x^2 + 2$.
- $-3(2x^3 - 4x^2 + 1) = -6x^3 + 12x^2 - 3$.

More examples

- $(-2x^3 - 5x + 4) + (3x^2 + 2x - 4) = -2x^3 + 3x^2 - 3x$.
- $(2x^2 + 3x - 2) - 2(-3x^3 - x^2 + 1) = 6x^3 + 4x^2 + 3x - 4$.
- $2(x^3 - 3x^2 + 4) - 3(-2x^2 - x + 3) = 2x^3 + 3x - 1$.

Adding and subtracting polynomials with two variables

- $2(3x^2 + xy + 2y + 1) - (x^2 - 2y^2 + 3x - 4y) = 5x^2 + 2y^2 + 2xy - 3x + 8y + 2$.
- $(x^2 + 3xy + 4x - y) + 2(4y^2 - 3xy - 2x + 2) = x^2 + 8y^2 - 3xy - y + 4$.