

13: Multiplying and Dividing Rational Expressions

Multiplying rational expressions

To multiply two fractions, we simply multiply the numerators and multiply the denominators. For example, $\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}$.

We do the same thing when multiplying rational expressions. For example, $\frac{2x^2+3}{-3x+1} \times \frac{x-4}{x^2+2} = \frac{(2x^2+3)(x-4)}{(-3x+1)(x^2+2)} = \frac{2x^3-8x^2+3x-12}{-3x^3+x^2-6x+2}$.

The resulting rational expression is not defined when $x = \frac{1}{3}$ and cannot be simplified.

Dividing rational expressions

To divide a fraction, a/b , by another fraction, c/d , we multiply a/b by the reciprocal of c/d , i.e., d/c . For example, $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{2 \times 7}{3 \times 5} = \frac{14}{15}$.

We do the same thing when dividing rational expressions. For example, $\frac{2x^2+3}{-3x+1} \div \frac{x-4}{x^2+2} = \frac{2x^2+3}{-3x+1} \times \frac{x^2+2}{x-4} = \frac{2x^4+7x^2+6}{-3x^2+13x-4}$.

The resulting rational expression is not defined when $x = \frac{1}{3}$ or $x = 4$ and cannot be simplified.

Simplifying using factoring

It's nice if the rational expressions we end up with can be simplified. It's generally easier to do this if we can factor any of the polynomials before multiplying. For example, $\frac{x^2+x-6}{x-1} \times \frac{x^2-2x+1}{x^2-4} = \frac{x^4-x^3-7x^2+13x-6}{x^3-x^2-4x+4}$, $\{\forall x \neq -2, 1, 2\}$. This is not particularly easy to simplify from here.

However, if we factor the numerators and denominators of the original rational expressions before multiplying, we get a much simpler answer: $\frac{x^2+x-6}{x-1} \times \frac{x^2-2x+1}{x^2-4} = \frac{(x+3)(x-2)}{x-1} \times \frac{(x-1)^2}{(x+2)(x-2)} = \frac{(x+3)(x-1)}{x+2}$.

More examples

- $\frac{x^2}{x^2-6x+9} \div \frac{4x}{2x-6} = \frac{x^2}{x^2-6x+9} \times \frac{2x-6}{4x} = \frac{x^2}{(x-3)^2} \times \frac{2(x-3)}{4x} = \frac{x}{2(x-3)}, \{\forall x \neq 0, 3\}.$
- $\frac{x^2-x-6}{x^3+2x^2} \times \frac{x^2+2x+1}{2x^2-4x-6} = \frac{(x+2)(x-3)}{x^2(x+2)} \times \frac{(x+1)^2}{2(x-3)(x+1)} = \frac{x+1}{2x^2}, \{\forall x \neq -2, -1, 0, 3\}.$
- $\frac{x^3+3x^2+3x+9}{x^2-4} \div \frac{x^2+2x-3}{x^2+x-2} = \frac{(x^2+3)(x+3)}{(x+2)(x-2)} \times \frac{(x+2)(x-1)}{(x+3)(x-1)} = \frac{x^2+3}{x-2}, \{\forall x \neq -3, -2, 1, 2\}.$