

15: Simplifying Complicated Expressions

Simplifying expressions to allow us to determine what happens to the expression as $h \rightarrow 0$

Example 1

$$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x + h, \{\forall h \neq 0\}.$$

This expression $\rightarrow 2x$ as $h \rightarrow 0$.

Example 2

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) = \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}}, \{\forall h \neq 0\} \text{ (note use of the conjugate).}$$

This expression $\rightarrow \frac{1}{2\sqrt{x}}$ as $h \rightarrow 0$.

Example 3

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\frac{x - (x+h)}{(x+h)x}}{h} = \frac{-h}{h(x+h)x} = -\frac{1}{(x+h)x}, \{\forall h \neq 0\}.$$

This expression $\rightarrow -\frac{1}{x^2}$ as $h \rightarrow 0$.

Simplifying expressions to allow us to determine what happens to the expression as $x \rightarrow a$

Example 4

$$\frac{\frac{3}{x+2} - \frac{3}{a+2}}{x-a} = \frac{\frac{3}{x+2} - \frac{3}{a+2}}{x-a} = \frac{\frac{3a+6 - (3x+6)}{(x+2)(a+2)}}{x-a} = \frac{3(a-x)}{(x-a)(x+2)(a+2)} = -\frac{3}{(x+2)(a+2)}, \{\forall x \neq a\}.$$

This expression $\rightarrow -\frac{3}{(a+2)^2}$ as $x \rightarrow a$.

Example 5

$$\frac{\frac{x+2}{x} - \frac{a+2}{a}}{x-a} = \frac{\frac{x+2}{x} - \frac{a+2}{a}}{x-a} = \frac{\frac{ax+2a - (ax+2x)}{ax}}{x-a} = \frac{2(a-x)}{(x-a)ax} = -\frac{2}{ax}, \{\forall x \neq a\}.$$

This expression $\rightarrow -\frac{2}{a^2}$ as $x \rightarrow a$.

Simplifying expressions to allow us to determine when the expression is 0

Example 6

$$\frac{4x^2(x-3)^{1/2} - x^3\left(\frac{1}{2}\right)(x-3)^{-1/2}(4)}{\left((x-3)^{1/2}\right)^3} = \frac{4x^2(x-3)^{1/2} - 2x^3(x-3)^{-1/2}}{(x-3)^{3/2}} = \frac{\frac{4x^2(x-3) - 2x^3}{(x-3)^{1/2}}}{(x-3)^{3/2}} = \frac{2x^3 - 12x^2}{(x-3)^2} = \frac{2x^2(x-6)}{(x-3)^2}, \{\forall x > 3\}.$$

This expression is 0 when $x = 6$.

Example 7

$$\frac{(3x+2)^{1/2}(2x+3)^{-1}(2)-(2x+3)\left(\frac{1}{2}\right)(3x+2)^{-1/2}}{(3x+2)^{1/2}} = \frac{\frac{2(3x+2)^{1/2}}{2x+3} - \frac{2x+3}{2(3x+2)^{1/2}}}{(3x+2)^{1/2}} = \frac{\frac{4(3x+2)-(2x+3)^2}{2(2x+3)(3x+2)^{1/2}}}{(3x+2)^{1/2}} =$$
$$-\frac{12x+8-4x^2-12x-9}{2(2x+3)(3x+2)} = -\frac{4x^2+1}{2(2x+3)(3x+2)}, \left\{ \forall x > -\frac{2}{3} \right\}.$$

There are no values of x for which this expression is equal to 0.