Adding Subtracting Rational Expressions

Transcript

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00:00:00:00 - 00:00:53:82
Narrator: Hello, and welcome to video number 14 in this series. In this video, I'll apply what we know about adding and subtracting fractions to adding and subtracting rational expressions. Way back in video number five, we reviewed adding and subtracting fractions. So for example, three fifths plus one third, and we need to find common denominators for this. An easy way to do that is to multiply each fraction by another fraction that's equivalent to one, and for the first fraction, we need to use the denominator of the other fraction, 3/3, and for the second fraction, we need to use the denominator for the first fraction.

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Narrator: 5/5. And that way, we end up with a common denominator of 15. We end up with nine plus 5/15, which is equal to 14/15. We apply essentially the same idea when we're adding or subtracting rational expressions. As an example, if we have two X plus one and we want to add X over X minus two, we just write it down again and leave a little bit of space like this.

00:01:39:20 - 00:02:38:21
Narrator: And we need to have two fractions here that are equivalent to one that will allow us to work with common denominators. So for the first fraction or the first rational expression rather, we look at the denominator of the other rational expression, X minus two, for the second rational expression, we look at the denominator for the first rational expression. Then now we've got common denominators. In the numerator, we'll have two times X minus two, two X minus four. Then for the next part, we've got X plus one times X, so we've got an X squared and an X, then then the denominator, we'll have X plus one times X minus two.

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Narrator: Then we can simplify the numerator here. We'll have an X squared, and let's see three X minus four, and that's a quadratic. Anytime we see a quadratic, we're always trying to factor it if we can. In this case, we need two numbers and multiply together to give us

minus four and then add to give us three. I think let's see, four and a minus one would work there.

00:03:11:51 - 00:03:42:84
Narrator: X plus four. And X minus one. That's a four and in the denominator, X plus one times X minus two. As always with rational expressions, we need to note any values of X that aren't permitted in the domain. In this case, that would be X not equal to minus one and two.

00:03:44:36 - 00:04:29:94
Narrator: Subtraction works very similarly. We still need to find common denominators. So let's do an example of subtracting rational expressions. Let's do X over four x minus one minus one over X plus two. So we'll have X over four x minus one times X plus two over X plus two and then we'll subtract X Sorry, four x -1/4 x minus one times one over X plus two.

00:04:32:02 - 00:05:47:37
Narrator: In the numerator, we're going to have X squared and two X minus four x and then plus one. And then denominator, four x minus one times X plus two. So in the numerator, we got X squared plus two X minus four x and minus two X and then plus one and that factors into X minus one squared in the denominator, four x minus one, X plus two, that's valid for all X not equal to minus two or one quarter. As always, when we're working with rational expressions, it's always nice if we can simplify the final answer or use factoring along the way to simplify the calculations. So let's see an example of how factoring might simplify things.

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Narrator: Let's do four over X squared minus two X plus one and add three over X minus one. If we proceeded with this just going through the motions and not thinking along the way, we would end up doing the following. Write down the first expression, multiply by X minus one over X minus one, and then write down the second expression and multiply by I haven't left myself enough room, but X squared minus two x plus one over X squared minus two x plus one. Then we're going to have to do all those calculations and we'll end up with in the numerator, four X minus four, plus three X squared, minus six X plus three. All divided by it's a lot.

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Narrator: It's complicated and it's messy and it turns out that it's too complicated in this case, because what we should have done is we shouldn't have just proceeded through the steps without thinking. We should have thought first this quadratic in this first rational expression in the denominator, that factors and in particular, it factors into x minus one squared. So just by doing that first step, we've made the entire problem much easier to solve because now we can see that the common denominator here is X minus one squared. This already has the common denominator in its denominator, there's nothing we need to do to that first expression. All we need to do is re express this rational expression with the common denominator.

00:08:20:27 - 00:09:22:73
Narrator: That just means multiplying it by X minus one over X minus. And then now in the denominator, we've got X minus one squared in the numerator, we'll have four and three X minus three. We can go one more step and say this is equal to four minus three is one, we'll have three X plus one. In the numerator and X minus one squared in the denominator, and that's valid for all X not equal to one. As another example, let's do X plus 1/2 X.

00:09:22:77 - 00:10:17:44
Narrator: This time, let's subtract the rational expression four over X squared plus three X. Again, we want to be thinking as we're moving through the steps and looking for opportunities to factor and we can see right away that the denominator in that second rational expression can be factored. Let's start there. X plus 1/2 X minus four over a common factor here is going to be X and we'll be left with X plus three. Now, the common denominator for these two will be two X X plus three.

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Narrator: X plus 1/2 X times X plus three over X plus three, and we're subtracting four over X, X plus three times. The common factor is going to be two X, X plus three, so I have to multiply by 2/2. We'll have in the numerator, X squared and X and three X and four X and three minus eight. Then in the denominator, we've got two X, X plus three. Let's see if that quadratic in the numerator will factor.

00:11:27:70 - 00:12:04:89
Narrator: We're going to have X squared plus four X minus five. We need two numbers and multiply to give us minus five and add to give us plus four. I think plus five minus one would work. Then in the denominator, two X, X plus three. And this is valid for all X not equal two minus three and zero.

00:12:05:86 - 00:13:04:23
Narrator: So pause the video at this point and see if you can work through the next two problems and use factoring where you can and try and do the addition or subtraction and simplify your final answer and also make a note of any values in the domain that are permitted. The first problem that I want you to work on is one over X plus two. Plus X over X squared plus three X plus two. Then the second one I want you to work on is X minus one over X squared plus X. We'll do a subtraction this time.

00:13:04:23 - 00:14:02:39
Narrator: We'll do X -1/2 X squared. Hopefully, you found the following two results. For the first one, first thing we're going to notice is that quadratic in the second rational expression, we can factor that because we just have to find two numbers that multiply together to give us two and add together to plus three. So the two numbers that work there would be plus two and plus one. Then we've got a common denominator of X plus two times X plus one.

00:14:02:39 - 00:14:46:30
Narrator: It's nothing we need to do to the second rational expression. We just need to get the first rational expression to have that common denominator, we'll multiply by X plus one over X plus one. We'll have one over X plus two times X plus one over X plus one and then X. X plus two, X plus one. Then finally, we just need to figure out what's in the numerator.

00:14:46:30 - 00:15:31:44
Narrator: We're going to have X plus one plus X. That's going to be two X plus one. Then in the denominator, we've got a X plus two X plus one. That's good for all X not equal to minus two or minus one. Then for the second one, we can factor the denominator of the first expression, pull out an X.

00:15:38:76 - 00:16:35:89
Narrator: So our common denominator is going to be x2x squared X plus one. So the first expression, we're going to have to multiply by two X. Then the second expression, we'll multiply that by X plus one over X plus one. And then figure out what's in the numerator and hopefully it'll simplify. We'll have two X times X, X squared, minus two X.

00:16:35:93 - 00:17:14:68
Narrator: Then here we've got minus X squared. And we've got plus X minus X. That's going to cancel out and we'll just be left with minus one minus minus one plus one. Then in the denominator, we've got two X squared X plus one. Let's see if we can factor that quadratic in the numerator.

00:17:14:68 - 00:17:49:59
Narrator: We've got two X squared minus X squared, so we got X squared minus X plus one. That's X minus one squared. Then we got X squared X plus one in the denominator. That's good for all X except four minus one and zero. That's all for this video.

00:17:49:59 - 00:18:03:95
Narrator: In the next video, we're going to bring together a lot of the ideas from the last few videos and see how that helps us simplify complicated expressions, more complicated than these ones.