Dividing Polynomials and Simplifying Rational Expressions

Transcript

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00:00:00:04 - 00:00:39:28
Narrator: Hello, and welcome to video number 12 in this series. And in this video, I'll talk about how we obtain a rational expression when we divide one polynomial by another and how we can sometimes simplify the resulting rational expression. So in video number ten in this series, we talked about multiplying polynomials to get another polynomial. So as an example, if we take two X squared plus three and multiply it by X minus four. So I'm multiplying two polynomials together.

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Narrator: I obtain two X cubed minus eight X squared plus three X -12, and that's another polynomial. But if instead of multiplying these two polynomials together, we divide them. So two X squared plus three divided by X minus four, then we're no longer in the family of polynomials because this is not a polynomial. And the name of this type of expression is it's called a rational expression, one polynomial divided by another. In this case, we can't simplify this, so that's it.

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Narrator: Another thing to note about rational expressions is that we can't have the denominator equal to zero because then the expression is not defined. So we always have to anytime we're dealing with a rational expression is make sure that there's a note somewhere to say that this is only defined for all X. So this upside down A is a symbol that stands for all. So for all X not equal to four, because if X was equal to four, this would not be defined. So let's go through a few more examples of rational expressions, but we'll work with rational expressions where we can actually do a little bit of simplification using ideas about factoring that we explored in the last video.

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Narrator: So here's another example of a rational expression, two X cubed, and six x squared over two x squared. And what we want to try and do is try and simplify this by

pulling out any common factors. So let's factor the polynomial that's in the numerator, and then maybe there'll be some canceling that we can do. So a common factor for the polynomial in the numerator here would be two x squared. And then I would need to multiply by X to get the first term and by three to get the second term.

00:03:20:48 - 00:04:11:91
Narrator: And we can see now that we've got two X squared in the numerator and two X squared in the denominator. So we can cancel those, and we'll end up with just X plus three. So in this case, we actually did end up with a polynomial. But actually, we didn't because we have to remember this notion of it's not defined when the denominator is zero. So this, we have to but a qualification that it's only defined for all X not equal to the only value of X that makes this zero would be zero itself.

00:04:11:91 - 00:04:53:47
Narrator: So this resulted in X plus three for any value of X except for x equals zero. So that actually makes this not a polynomial because polynomials don't have those restrictions. Let's do another example. Let's do X squared minus four, and X squared plus three X minus ten. So what I'm going to do is I'm going to see if I can factor the numerator and the denominator and see if we can get any canceling going on.

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Narrator: We should recognize this as perfect square minus another perfect square, so we can factor this using the idea of conjugates. So X minus two, X plus two. And then the denominator, if we can find two numbers that multiply to give me minus ten and add to give me plus three, I can factor this, and let's see, plus five, X minus two works. And then I can see I've got a common factor in the numerator and the denominator, so I can cancel those. I end up with X plus two over X plus five.

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Narrator: But this is only valid for Xs where the denominator is not equal to zero, so I'm not going to be able to use x equal minus five or x equal plus two. Let's do another example. Let's do X squared plus X -12, and X minus three, all squared. See if I can factor the numerator here. Two numbers that multiply to give me -12 and add to give me plus one.

00:06:28:60 - 00:07:14:42
Narrator: So plus four and minus three would work, plus four minus three. And then we got X minus three squared in the denominator. So the X minus three in the numerator will cancel with one of those. X minus three is in the denominator, and we end up with X plus four over X minus three. And again, I just got to put my domain restriction here as long as X not equal to three.

00:07:19:24 - 00:08:02:18
Narrator: Let's do. Let's do a slightly more complicated example. Let's do X squared plus four x plus four times two X plus two divided by X squared minus one X plus two. Okay, so

see if I can factor that quadratic X squared plus four, X plus four. I need two numbers and multiply to give me four and add to give me four.

00:08:02:18 - 00:08:30:00
Narrator: Well, that's easy. X plus two times X plus two, X plus two squared. There's a common factor here for the two X plus two, a common factor of two. Then in the denominator, X squared minus one, this is a difference of perfect squares. So I can factor that using conjugates.

00:08:34:05 - 00:09:11:83
Narrator: And then let's see if I can do any canceling. One of the ex plus two is here could cancel with this one in the denominator. And the X plus one here cancel with the one in the denominator. So I've ended up with two X plus two over X minus one. Again, just put my domain restriction as long as X is not equal to one or minus one or minus two.

00:09:11:83 - 00:10:16:61
Narrator: X not equal to, and I'll just put those in increasing order minus two minus one and plus one. So thinking about the ways that we've done this, pause the video at this point and see if you can work through the next two problems by yourself and then restart the video and see if you get the same answers. So what we're trying to do is take a rational expression and simplify it by using factoring. So the first one, let's do three X cubed plus three X squared -18 x. Divide that by six X squared -12 X.

00:10:17:63 - 00:11:10:25
Narrator: And then the second one, let's do let's make it a little more challenging. 2 X minus six, three X squared plus two X, and then three X plus two and X squared minus nine. And so work through these two examples, simplify using the ideas of factoring, and also make a note of any values of X for which the rational function is not defined. So let's see how you did this first one. So let's see if we can factor the numerator, so I can pull out a three X.

00:11:10:95 - 00:11:44:61
Narrator: We'll have X squared plus X minus six. And then let's factor the denominator two. I can pull out six X. So I'll need an X and then a minus two. And see if I can factor that quadratic up there.

00:11:44:61 - 00:12:13:25
Narrator: X squared plus X minus six. I need two numbers that multiply to give me minus six and add to give me plus one. So let's see plus three minus two would work there. So I'll have three X, X plus three and X minus two. And then I've got six X, X minus two.

00:12:13:25 - 00:12:55:16
Narrator: And then let's see if I can do some canceling. So the X and the X would cancel, the X minus two and the X minus two would cancel, and we've got a three and in the numerator and a six in the denominator. So I could cancel that, and it will leave me with two in the denominator. So my final answer is X plus 3/2. And I just need to make sure that I'm noting down any values of X for which this rational expression is not defined.

00:12:55:16 - 00:13:28:46
Narrator: So it's not going to be defined when X is either zero or two. So for all X not equal to zero or two. And then the second one. Let's see. I can pull out a common factor of two for that first term.

00:13:28:46 - 00:14:03:23
Narrator: And then a common factor of let's see, X is all I can pull out of that second term. Three x plus two. And then in the denominator, I got three X plus two. Then this is a difference of squares. So I can factor that using conjugates, X minus three and X plus three.

00:14:03:83 - 00:14:41:34
Narrator: Let's see if I can do some canceling now. I got X minus three in the numerator and the denominator. I got three X plus two in the numerator and the denominator. So I end up with two X in the numerator, X plus three in the denominator, and this is valid for all X not equal to. So if X was equal to minus two thirds, this first term, this first factor, and the denominator would be zero.

00:14:41:34 - 00:15:20:17
Narrator: So that's not allowed. Or if X was plus three or minus three, then we would end up with zero in the denominator there, and the rational function would not be defined. So X cannot equal minus three and minus two thirds and plus three. Any other value of X is fine. So that's it for this video, and in the next video, we'll stay with rational expressions, and we'll look at what happens when we multiply and divide rational expressions.