Factoring Polynomials

Transcript

Instructor: Iain Pardoe

00:00:00:00 - 00:00:51:29
Narrator: Hello, and welcome to video number 11 in this video series. In this video, I'm going to provide some examples of factoring polynomials. In the last video, we talked about multiplying two polynomials together to get another polynomial. As a very simple example, we could multiply this polynomial to X squared by this polynomial, X minus four, and we end up with a new polynomial two X cubed minus eight X squared. The two x squared and the X minus four are called factors of two X cubes minus eight x squared.

00:00:51:37 - 00:01:50:35
Narrator: If we go from two x cubed minus eight x squared back to the two factors that multiply together to create this polynomial, that process is called factoring this polynomial. And that turns out to be an important thing to do as you move on through your math journey. The reason that it becomes important in certain areas of mathematics is often we're looking for where a polynomial is equal to zero. And by factoring the polynomial, we can easily see where this polynomial is equal to zero. In this case, this polynomial is equal to zero when either this factor is zero, so that would happen when X is zero or when this factor is zero, and that would happen when X is four.

00:01:51:51 - 00:02:40:16
Narrator: As another example, let's do three X cubed minus three x squared plus six X. Let's try and factor that polynomial. What we're looking for is a factor for each of the terms. A common factor for each of those terms so that we can pull out that factor and separate it. So what I can notice here is that in terms of the constant that's multiplying the constant coefficient that's multiplying each of these powers of X, we've got three.

00:02:40:16 - 00:03:00:40
Narrator: Three is a common factor. But also, we've got X cubed, X squared, and X. X is going to be a common factor. Three x is a common factor. Then I just have to figure out what I multiply three X by to get this polynomial.

00:03:00:40 - 00:03:47:33
Narrator: So to get the first term, I'm going to have to multiply three X by X squared. To get the second terms, I'm going to have to multiply by minus X, and for the third term, I need to multiply by two. I factored this polynomial we all three x cubed minus three X squared plus six X into two factors, three X and X squared minus X plus two. Degree two polynomiols are called quadratics. I'll spend a little bit of time just going through factoring quadratics, because that's a very common task that you'll need to perform.

00:03:47:33 - 00:04:19:39
Narrator: As an example, let's think about this quadratic. Let's do another color. Let's X squared plus five X plus six. And I want to factor that. The form of the two factors is going to be X plus some number, A, and then X plus some other number B.

00:04:20:84 - 00:05:02:83
Narrator: And I just need to figure out what A and B are. If we look at the form of this quadratic, I've got the coefficient for X square is just one, that's going to work out here because I've got X times X, that's okay. If I look at the constant term, that's going to come from multiplying A and B. A and B are going to have to multiply to get six. Then as far as the X term, if I think about the two factors here, I'm going to have A times X and also B times X.

00:05:03:23 - 00:05:34:91
Narrator: The sum of A plus B is going to have to equal five. I need to think of two numbers that multiply together to get six and add to get five. It's fairly easy to see that two numbers that work there are two and three. If we multiply this out, we're going to get this polynomial, this quadratic polynomial. Let's do another example.

00:05:34:91 - 00:06:16:41
Narrator: Let's do X squared plus three X minus ten. Again, we're trying to find two numbers that in this case, are going to multiply to give us minus ten and add to give us plus three. If I think about the factors of ten, I've got one and ten, I've got two and five. But I'm also thinking about here, I've got to get them to add together to get three. I think two and five ought to work because one of them is going to have to be negative.

00:06:16:41 - 00:07:40:42
Narrator: If I'm going to end up with plus three here, I'm going to need plus five and minus two because five times minus two is minus ten and then five minus two is three. This is an important skill to develop the ability to go from a quadratic into its factors, part of the motivation for that is we're often looking for where the polynomial is zero and we can easily see, for example, in this case, this polynomial would be zero when X is equal to minus five or X is equal to plus two. In the last video, we did an example where we multiplied four x plus three and four x minus three. I said this was the conjugate of this, four x minus three is the conjugate of four x plus three. What was interesting about multiplying these two polynomiols is that we get the partial product from multiplying four x and four x.

00:07:41:42 - 00:08:37:96
Narrator: And then we get the partial product from multiplying plus three and minus three. But the other two partial products cancel each other out because we've got plus 12 X for multiplying these two and -12 X from multiplying these two. Whenever we see a polynomial that looks like this, there's a term here, which is a perfect square and another term here, which is also a perfect square, then we can factor it using the idea of conjugates. For example, if I want to find the factors for nine x squared minus four, I've got a perfect square here, which will be three X because if I take three X and I square it, I get nine X squared. I got a perfect square here because two squared is four.

00:08:37:96 - 00:09:17:29
Narrator: I know that I can factor this as three X minus two, and three X plus two. I wrote it round with minus here and the plus. It really doesn't matter which order you write the factors in. But just checking, we did the right thing three X times three X is nine X squared and minus two times plus two is minus four. Then the other two partial products cancel out, it's got minus six x and plus six x.

00:09:18:10 - 00:09:54:19
Narrator: Let's do another example, let's do four x squared -25. That's going to be 2 X minus five, two X plus five. Because the two X is combined to make four X squared and the five's combined to make 25. Let's do another related example. Let's do X squared minus five.

00:09:54:43 - 00:11:02:92
Narrator: In this case, this is a perfect square, but five is not a perfect square. There's no whole number that we can multiply together to get five. But if we expand our number system that we're working with into all real numbers, not just rational numbers, but irrational numbers as well, then we could factor this again using the idea of conjugates as X minus the square root of five and X plus the square root of five. And just checking that we did the right thing, we're going to have X times X gives us the X squared. Then we've got minus root five times plus root five, multiply those together, and we'll get minus five, and then got minus root five times X, which will cancel with plus root five times X.

00:11:05:41 - 00:11:48:18
Narrator: Then the other thing we can think about is here we've just got one variable X, but we could also think about factoring when we've got more than one variable. As an example, let's do 6x cubed, Y -15 X, Y squared. Let's try and factor that. So when we're factoring, we're looking for a common factor for each term. In terms of the coefficient in front, the six and the 15, a common factor there would be three.

00:11:48:23 - 00:12:15:17
Narrator: And in terms of the X cubed Y and xy squared, a common factor would be XY. X Y is my common factor, and then what do I need to multiply? The X Y by to get six X cubed Y. I'll need to multiply by two X squared. Then what do I need to multiply?

00:12:15:17 - 00:13:15:67
Narrator: Three XY, to -15, xy squared, I'd need minus five and then Y. Let's work through a few more examples. Let's have two X, X minus four and three X minus four. Remember, we're trying to find common factors for each of these terms and we can see right away that X minus four is a common factor. I'm actually going to write the common factor with a space because I can see that two X I can pull out of the first term and then three, I can pull out of the second term.

00:13:17:91 - 00:14:22:39
Narrator: Second example, let's do two x cubed x minus three and two X, X minus three squared. Looking for common factors, two X, X minus three. Then I need to multiply by X squared for the first term and by X minus three for the second term. That's as far as I go, I can't factor this any further. On third example, let's do two x cubed minus two x squared, -24 X.

00:14:25:03 - 00:15:21:12
Narrator: A common factor here would be two X, then we'll have X squared minus X -12. Then I can actually go one more step with this one because I can factor this because I need two numbers that multiply to give me -12 plus three minus four would do that. When I add them, I need to get minus one. So two X, X minus four X plus three, check. Anytime you factor a quadratic, always check that you've done it correctly.

00:15:21:12 - 00:16:22:86
Narrator: For the constant term, I've gone minus four times plus three, which should be -12. Then for the X term, I've gone minus four x plus three X, which does give me minus X. And then fourth example, let's do three X cubed Y -12 X Y cubed. A common factor here would be three X, Y, then the first term, I need another X squared, and then the second term, I need four Y squared. I can also go one more step here because I can recognize that this is in the form of a perfect square minus another perfect square.

00:16:22:86 - 00:17:20:42
Narrator: I can factor this using conjugates. It would be X minus 2Y and X plus 2Y. Again, every time you factor this polynomial using conjugates, just double check by going backwards to make sure that you did it correctly. I've got X squared and then I've got minus two y times plus two y, that'd be minus four y squared and then minus two XY plus two xy would cancel out. At this point, pause the video and work on the next two problems that I'll give you and see if you can come up with the answers yourself and then start the video again and review if you got them correct or not.

00:17:20:42 - 00:18:10:30
Narrator: For the first problem, let's do three X squared plus nine X -12. See if you can factor this. For the second one, let's do X cubed plus two X squared. Minus nine X -18. I'm going to give you a hint for this second one because sometimes it can be a little bit difficult to get going with these problems.

00:18:11:06 - 00:18:50:07
Narrator: I'll give you a little hint with this one. If we regroup this and write it in this form, combine the X cube with the minus nine X and then the plus two X squared -18. Then that should get you going with this one. For this first one, first thing we can do is pull out a common factor of three. We'll have three X squared plus three X minus four.

00:18:51:37 - 00:19:29:31
Narrator: Then we can see if we can factor this quadratic if we can find two numbers that multiply together to give me minus four and then add to give me plus three. I think let's see plus four minus one would work because they would multiply to give me minus four and add to give me three. I'm going to have three and then X will have plus four. And X minus one. Just double check.

00:19:29:31 - 00:20:00:76
Narrator: We'll go backwards, make sure we get what we started with. We'll have X squared and then we'll have four X minus X, we'll have plus three X, and we got four minus one times minus one minus four. And then multiply everything by three and we'll be back where we started. I think this is correct. Then with this one, I gave you the hint of rearranging this into Xcubed minus nine X.

00:20:00:76 - 00:20:51:19
Narrator: The reason I wanted to do that was because I'm going to pull out a common factor of X here and be left with X squared minus nine. I'm going to pull out a factor of two here, and then I'll have X squared minus nine. Then now I've got a common factor of X squared minus nine. I'm going to have X plus two times X squared minus nine. Then I can recognize that this is in the form of a perfect square minus another perfect square.

00:20:51:19 - 00:21:24:12
Narrator: I can factor this using conjugates. I'll have X plus two and then minus three X plus three. Again, we'll just check we did the right thing by going backwards. X minus three times X plus three would give me X squared minus nine. That's this piece, and then the X plus two.

00:21:24:12 - 00:21:55:61
Narrator: Then I can separate this into an X times X squared minus nine and a two times X squared minus nine, which is this step, and then that gets me back to where I started. That's all for this video. In the next video, I'll finish off working with polynomials by looking at what happens when you divide one polynomial by another polynomial and that leads to a new expression called a rational expression.