



Multiplying and Dividing Fractions

Transcript

Instructor: Iain Pardoe

00:00:00:00 - 00:01:02:19

Instructor: Hello, and welcome to video number four in this series. In this video, I'll discuss multiplying fractions and I'll show how a connection between multiplication and division provides a rationale for dividing fractions. So, to multiply two fractions A over B and C over D , we just multiply the two numerators, which is the A and the C . And then we multiply the two denominators, the B and the D and the result is A times C divided by B times D . For example, two thirds times five sevenths is equal to two times five in the numerator, which is ten and three times seven in the denominator, which is 21.

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Instructor: The answer is $10/21$. Multiplying fractions together, it's an easy thing to do. We just multiply the numerators and multiply the denominators. But why does it work? We can think about multiplication as representing two thirds of five sevenths.

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Instructor: If we think about it like that, I could imagine a unit square, and if I divide the height of it into one third, two thirds. I've divided it into thirds and I've sectioned off two thirds and then along the width, I'll divide this into seven pieces, one, two, three, four, five, six, and the last one is seven, and I'll mark off five of those. Five sevenths. There and then I'll draw horizontal lines at the thirds and vertical lines at the seventh and then I'll shade in the piece representing two thirds times five sevenths. That's going to be those rectangles there.

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Instructor: And how many rectangles have I shaded? So I've got five. I've got two times five. I've got ten. Well, that reminds me of something.

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Instructor: That was what this was up here, two times five. Then how many rectangles do I have in total? I've got three times seven. Three times seven, that's what I did up here. The part that I've shaded represents the numerator.

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Instructor: I've got ten parts that are shaded out of 21 parts in total. This provides a graphical interpretation of how fraction multiplication works. Here's another example, two thirds times five eighths. We'll just apply our method from before. In the numerator, I'll have two times five, ten.

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Instructor: In the denominator, I'll have three times eight. 24. If we think back to the last video, this is not in lowest terms. So let's get it into lowest terms. There's a common factor in ten and 24 of four.

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Instructor: No. $10/24$, it's not in lowest terms, thinking back to the last video, let's reduce this to lowest terms. Think of a common factor for ten and 24. Two is a common factor here, $10/2$ is 5 and $24/2$ is 12. The answer here in lowest terms is $5/12$.

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Instructor: There's a way of getting to $5/12$ that is sometimes a little bit quicker. Let me show you how to do that. It's to think about these fractions that we're multiplying first and see if we can do any factor reducing here before we get to here. I'll just write it down again. Think about our method for reducing to lowest terms.

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Instructor: We're looking for a common factor in the numerator and the denominator. Because I'm just multiplying the numerators here and multiplying the denominators here, the common factor needn't be within the same fraction, it might be across to the other fraction. Here there's a common factor for the two and the eight, which is two. $2/2$ is 1 and $8/2$ is four. Now, when I multiply my numerators, I've got one times five, which is five and I've got three times four now, which is 12.

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Instructor: This is one of those calculation shortcuts that helps you move through calculations a little bit quicker. This technique of pulling out common factors with fractions that we're multiplying will come up again when we're multiplying more complicated expressions, not just fractions, but later on we'll talk about rational expressions which have more complicated algebra to them, but the idea of pulling out common factors is the same. So that's fraction multiplication, which is relatively straightforward. Move on to fraction division now, which is a little bit more complicated, but if we build a connection between multiplication and division, it's actually no more complicated than this. We just have to do one extra step.

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Instructor: Before we think about dividing fractions, let's first think about how we divide just integers. For example, A divided by B. One way we could think about this is we could turn it into a multiplication problem, which is often easier to think about than a division

problem. So instead of dividing A by B, I'm going to multiply A by the reciprocal of B. These two things are equivalent.

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Instructor: A divided by B is the same as A times one over B as long as B is not zero. For example, $\frac{3}{4}$ is the same as three times $\frac{1}{4}$. It's just three quarters or 0.75. Let's think about fractions now.

00:08:15:92 - 00:08:41:12

Instructor: Fraction A over B, divided by C over D. Well, let's just do the same thing as we did here. Let's turn this into a multiplication problem. What did I do? I had my A, I just left that as it was, and then I multiplied by the reciprocal of the thing I was dividing by.

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Instructor: What's the reciprocal of C over D? Well, you just flip it the other way up. D over C. And then we know how to multiply fractions. I'll be A, D over B, C.

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Instructor: For example, two thirds divided by five sevenths. That's going to be equal to two thirds times the reciprocal of five sevenths, which is $\frac{7}{5}$. And then two thirds times $\frac{7}{5}$, be two times seven in the numerator and three times five in the denominator. We'll end up with $\frac{14}{15}$. Let's run through a few more examples.

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Instructor: Let's do two thirds divided by $\frac{5}{8}$. Two thirds times the reciprocal of five eighths, which is $\frac{8}{5}$. Two times eight in the numerator, 16 and three times five in the denominator, which is 15. That's an improper fraction, so we could go one more step and express that as 1 and $\frac{1}{15}$. Let's do five eighths divided by two thirds.

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Instructor: We'll start with five eighths, multiply by the reciprocal of two thirds, which is $\frac{3}{2}$. Now we have five times three in the numerator and eight times two in the denominator, 16. You'll see from these two examples that two thirds divided by five eighths is not the same thing as five eighths divided by two thirds. But you can see these are related. This is the reciprocal of this, this is the reciprocal of this.

00:11:29:39 - 00:11:58:78

Instructor: Let's do another example. A good. Okay. Let's do another example. Let's do five eighths divided by three quarters.

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Instructor: Five eighths times reciprocal of three quarters, which is $\frac{4}{3}$. So five times four is 20, eight times three is 24. This is not in lowest terms. There's a common factor of four in the numerator and the denominator. So four times five comes to 20 and 4 times six comes to 24.

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Instructor: We could have got to this a little bit quicker if we apply something we talked about over here, which is looking for common factors when we're multiplying fractions across the two fractions. Because here, I've got five eighths times four thirds. If I look for common factors in the five eighths and the four thirds, there's a common factor in the eight and the four. There's two fours in eight and one 4 in four. Now we've got five times one in the numerator and two times three in the denominator.

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Instructor: So whenever you're doing these calculations and manipulations, always look for shortcuts, just to save yourself a little bit of time. Pause the video at this point and see if you can do the next two fraction divisions, and again, try to resist the temptation to use a calculator and see if you can just work through the methods that we've used over here, pulling out common factors to speed things up if you can and try to get the answers without using a calculator. So let's do four ninth divided by five sixth and that's the first one and then the second one, let's do five ninths, divided by $4/15$. Four ninths divided by $5/6$, so four ninths times, and then the reciprocal of five sixth will be $6/5$, and I can right away see a common factor here for the nine and the six. There'll be three threes in nine and two threes in six.

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Instructor: Now I'll have four times two, which is eight and three times 5, 15. Then for the second one, we'll have $5/9$ times $15/4$. I can see a common factor for the nine and the 15. There'll be three threes in nine and five threes in 15. Now we'll have five times five is 25 in the numerator and three times 4, 12 in the denominator.

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Instructor: That's an improper fraction. Let's express it as a mixed number. 12 goes in 25, two times, will be 24 and then there'll be one more, two and one 12th. That's it for this video. In the next video, I'll continue working with fractions and we'll think about fraction, addition and subtraction, which turns out to actually be a little more complicated than multiplication and division.