



## Multiplying and Dividing Rational Expressions

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### Transcript

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**Narrator:** Hello, and welcome to video number 13 in this series. And in this video, I'll demonstrate how to apply what we know about multiplying and dividing fractions to multiplying and dividing rational expressions. So in video number four in this series, we went through some examples of multiplying fractions. For example, two thirds times five sevenths. And we noted that all we do is multiply the numerators and multiply the denominators.

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**Narrator:** So we got two times  $\frac{5}{3}$  times seven, which is  $\frac{10}{21}$ . So we do something similar if we're multiplying rational expressions. For example, if I have this rational expression, two  $X$  squared, plus three divided by minus three  $X$  plus one, and I want to multiply that rational expression by  $X$  minus four over  $X$  squared plus two. I'll actually use a dot here to signify multiplication so it doesn't get mixed up with  $X$ s. And so all we do is same thing as we did with fractions, multiply the numerators and then multiply the denominators.

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**Narrator:** So multiplying the numerators, I'm going to have two  $X$  squared times  $X$ . So that's going to be two  $X$  cubed. I'll have two  $X$  squared times minus four, so minus eight  $X$  squared. I have plus three times  $X$ , and I'll have plus three times minus four. And then in the denominator, I've got minus three  $X$  times  $X$  squared, so minus three  $X$  cubed.

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**Narrator:** And then I've got minus three  $X$  times two, so minus six  $X$  plus one times  $X$  squared. And then plus one times two. Okay, so I've ended up with another rational expression. And in this case, we can't simplify this one any further. And just as in the last video, we always need to make sure that our domain is defined when we're dealing with rational expressions.

00:02:58:71 - 00:03:34:64

**Narrator:** And in particular, we can't have the denominator be equal to zero. And so in this case, the denominator would be zero if  $X$  was minus one third. And actually, that's it because this part  $X$  squared plus two, that is always positive, so that can't be zero. So what we have to do is say, for all  $X$  not equal to minus one third. Dividing rational expressions also works the same as dividing fractions.

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**Narrator:** So if we go back to our first example with the fractions, let's do two thirds divided by five sevenths. So to do this, we leave the first fraction as it is, change the divide to multiply, and again, I'll use the dot and then flip the second fraction. This is the reciprocal of this fraction. And then apply what we've already done with multiplying fractions, multiply the numerators, multiply the denominators. So we'll have  $14/15$  in this case.

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**Narrator:** Let's do the same idea using the rational expressions. Let's do two  $X$  squared plus three minus three  $x$  plus one, and let's divide by  $X$  minus four  $X$  squared plus two. So leave the first rational expression as it is. And then the second one, we turn it upside down. We take the reciprocal.

00:05:12:40 - 00:06:08:68

**Narrator:** And then we just proceed as we did for the last example. So we'll have in the numerator, let's see, we'll have two  $X$  to the four, and we'll have a three  $X$  squared and four  $X$  squared, seven  $X$  squared. And we'll have a plus six, and in the denominator, we'll have minus three  $X$  squared, and we'll have plus  $X$  plus another 12  $X$  or 13  $X$ . And we'll have minus four. And this is defined for all  $X$  not equal to the denominator could be zero whenever  $X$  is equal to minus one third or  $X$  is equal to four.

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**Narrator:** And so let's just work through a couple more examples. And in the next two examples that I'm going to work through. It's always nice if we can simplify our algebra. And so we're going to use the idea of factoring to see if we can simplify the rational expression that we end up with. So Let's do this example, Let's do  $X$  squared plus  $X$  minus six for  $X$  minus one times  $X$  squared minus two  $X$  plus one, and then  $X$  squared minus four.

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**Narrator:** So I'm going to multiply the numerators and multiply the denominators. So I'll have  $X$  to the four and minus two  $X$  cubed plus one  $X$  cubed minus  $X$  cubed. And then we'll have minus two  $X$  squared, minus six  $X$  squared. We've got minus eight  $X$  squared, and then we'll have minus six. And then in the denominator, we've got an  $X$  cube, and we've got a minus  $X$  squared and we've got a minus four  $x$ , and we've got a plus four.

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**Narrator:** So we've ended up with a rational expression, and it's a little challenging to factor this. It's not impossible because it does actually factor. But let's let's think a little harder

about how to make this a little easier. What if we can factor back here? And then that will lead us to a simpler expression.

00:08:17:67 - 00:08:55:88

**Narrator:** So in this case, when you're trying to multiply rational expressions or divide them, if you can do any factoring before you start multiplying the rational expressions, the polynomials, you can end up simplifying the final expression you end up with. So let's see if we can do that in this case. So I'm going to kind of forget what I did here. And kind of start over. So let's see if we can factor  $X^2 + X - 6$ .

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**Narrator:** Can we factor that quadratic? We need two numbers and multiplied to give us minus six and add to give us plus one. So plus three minus two would work. And then let's see if we can factor this quadratic. So we need two numbers and multiply to give us plus one,  $A$  to give us minus two, so minus one would work there.

00:09:27:04 - 00:09:52:63

**Narrator:** Minus 1,  $X$  minus one all squared. And then this is a difference of squares. So I'm going to use conjugates here to factor that  $X$  minus two,  $X$  plus two. And so now we've got a bunch of factors in the numerator and a bunch of factors in the denominator. I can do some canceling here.

00:09:52:63 - 00:10:46:37

**Narrator:** I can cancel this  $X$  minus two with this one, and I can cancel one of these  $X$  minus ones with the one down here, and I've ended up with a fairly simple looking rational expression. I've got  $X$  plus three, and I've got  $X$  minus one, and then in the denominator, I've got  $X$  plus two. So this answer is much easier to deal with than this one. And so, just by thinking ahead and doing some factoring, I ended up with an easier final result. And so we just need to specify the values of  $X$  that we're not going to be permitted to use in the domain.

00:10:46:37 - 00:11:13:66

**Narrator:** And so we just have to look at what values of  $X$  would make the denominator zero. So we can have any value of  $X$  except for plus one or plus two or minus two. So  $X$  not equal to minus two or one or two. Let's do one more example. That's two.

00:11:13:66 - 00:11:44:27

**Narrator:** Let's do a division. Let's do  $X^2$  over  $X^2$ , minus six  $X$  plus nine. Let's divide by four  $X$  over two  $X$  minus six. Again, let's twin, do as much factoring as we can. Before we start multiplying the polynomials.

00:11:44:27 - 00:12:11:34

**Narrator:** So I'm gonna have  $X^2$ . And then see if I can factor this quadratic here. Two numbers and multiply give me plus nine and add to give me minus six. So I think minus three  $\times$  minus three squared would do that. And then I'm multiplying by.

00:12:11:34 - 00:12:52:00

**Narrator:** I've got to take the reciprocal of this second rational expression So I'll have two  $X$  minus six in the numerator, so I'll take a factor of two there. And then the denominator, I'll have four  $x$ . Oh, let's see if I can do some canceling. One of those  $X$  minus three is down here will cancel with this one up here. The  $X$  down here will cancel with one of those  $X$ s up there, and then the two and the four, I'll have two left in the denominator.

00:12:52:52 - 00:13:57:27

**Narrator:** Now I'm mended up with  $X$  in the numerator and two  $X$  minus three in the denominator. And this is good for all  $X$  not equal to anything that would make the denominator zero. So  $x$  equal three or  $x$  equal zero. Okay, so pause the video at this point and see if you can work with the next to rational expression, multiplication or division problems and try to do some factoring before you start multiplying polynomials and see if you can get to the correct answers. And again, note any values of  $X$  in the domain that are not permitted.

00:13:57:27 - 00:15:08:56

**Narrator:** So for the first example, let's do multiplication. Let's do  $X$  squared. Minus  $X$  minus six over  $X$  cubed plus two  $x$  squared. Let's multiply that rational expression by  $X$  squared plus two  $X$  plus  $\frac{1}{2}x$  squared minus four  $x$ , minus six, and then the second one, let's do  $X$  cubed plus three  $x$  squared plus three  $X$  plus nine divided by  $X$  squared, minus four, let's divide that by  $X$  squared plus two  $X$  minus three and  $X$  squared plus  $X$  minus two. Okay, so let's see how you did.

00:15:08:56 - 00:15:38:70

**Narrator:** This first one, let's see if I can factor this quadratic up here. Two numbers that multiplied to give you minus six and add to give me minus one. So  $X$  minus three  $X$  plus two should work. And then see if I can factor this quadratic, as well. Two numbers that multiply to give me plus one and add to give me plus two.

00:15:38:70 - 00:16:44:13

**Narrator:** So  $X$  plus one squared and then let me factor this polenomal here so I can pull out an  $X$  squared, and then I'll have  $X$  plus two and then see if I can factor this quadratic. Well, if I pull out two, then I'll be left with  $X$  squared minus two  $X$  minus three and two numbers multiply to give me minus three and add to give me minus two. So I think  $X$  minus three  $X$  plus one should work. Let's just check that  $X$  squared, minus three  $X$  plus  $X$  minus two  $X$ , and then minus three. I think that works.

00:16:44:13 - 00:17:18:01

**Narrator:** Let's let's do some canceling. Let's see. I got  $X$  minus three and  $X$  minus three,  $X$  plus two, and  $X$  plus two,  $X$  plus one squared and  $X$  plus one, one of those  $X$  plus ones would cancel on the numerator. And I'm left with  $X$  plus one. In the numerator and two  $X$  squared in the denominator.

00:17:18:01 - 00:17:58:41

**Narrator:** And this is valid for all  $X$  not equal to. There's a few values of  $X$  that I'm not permitted to use here, zero minus two plus three and minus one. Uh, let's try and write those in order. We're not allowed minus two and minus one and zero and three. I think I got them all. Okay, so that's the solution for

00:17:58:41 - 00:18:37:50

**Narrator:** the first problem and for the second one. Let's see. So let me try and factor this cubic expression. I do  $X$  squared and  $X$ . So I'm going to have two numbers here that multiply to give me nine, and I'm going to want a three  $X$  and a three  $x$  squared.

00:18:37:50 - 00:18:57:61

**Narrator:** So I think if I just put plus three here and here, I think that should work. I've got  $X$  cubed, I've got three  $X$  squared. I got three  $X$ , and I've got nine, so that works. And then I'm dividing by this rational expression. So I've got to take the reciprocal.

00:18:57:61 - 00:19:26:98

**Narrator:** I've got to flip it up upside down. So I've got  $X$  squared plus  $X$  minus two in the numerator, and I'm going to factor that if I can find two numbers that multiply to give me minus two and add to give me plus one. So I think  $X$  plus two  $X$  minus one would work. Because I've got the minus two, and then I've got plus two  $X$  minus  $X$ . I've got the plus  $X$  as well, so that's good.

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**Narrator:** And then in the denominator, I recognize this is a difference of squares. So I'll factor that using conjugates. And then remember I flipped the second rational expression upside down. So now the numerator is in the denominator, and I'm going to try and factor that quadratic. Two numbers that multiply to give me minus three and add to give me plus two would be  $X$  plus three  $X$  minus one.

00:20:01:62 - 00:20:28:87

**Narrator:** I think that's right, just check. I got plus three  $X$  minus  $X$ , I got plus two  $X$ , and I got plus three times minus one is minus three. Oh, yes, and now I'm going to do my canceling let's see, I got  $X$  plus three. I got  $X$  plus two, I got  $X$  minus one. So this simplifies quite a lot.

00:20:28:87 - 00:21:10:99

**Narrator:**  $X$  squared plus three in the numerator and  $X$  minus two in the denominator. And this is valid for all  $X$  not equal to, anything that makes the denominator zero. So minus three and minus two, and one and two. So that's it for this video. And in the next video, we'll finish up our exploration of rational expressions by looking at adding and subtracting them.