



Simplifying Complicated Expressions

Transcript

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00:00:00:00 - 00:00:51:52

Narrator: Hello, and welcome to video number 15 in this series. In this video, I'm going to review some examples of simplifying complicated expressions that use a lot of the ideas that we've looked at in previous videos. For a first example, I want to see if I can simplify this expression. The reason I want to try and simplify it is I'd like to know what happens to this expression as the value of H goes towards zero. I can't really do that in the form it's in at the moment because we've got H in the denominator and if H was zero, this expression wouldn't exist.

00:00:52:04 - 00:01:52:94

Narrator: Let's see if we can figure out what this expression might look like and then we might be able to determine what happens to it when H goes to zero. Just looking at the expression, I think the only thing I can do initially is expand what's in the numerator and then go from there. I have to expand X plus H squared, so I'll have X squared plus two xh plus H squared. Then I'm subtracting X squared and I'm dividing all that by one thing I notice is I've got an X squared there and a minus X squared there. Then the other thing I notice is what's left.

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Narrator: I've got a common factor of H in the numerator. I've got H times two X plus another H . Then I see that I have an H in the numerator and the denominator. This simplifies to two X plus h and now I can look at this expression and I see that as H goes towards zero, this expression goes towards two $2x$. I wouldn't have been able to figure that out just looking at this expression before I simplified it. Let's do a similar example where again, we're trying to figure out what the function does as H goes towards zero.

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Narrator: For this one, I'm going to bring in some square roots. Square root of X plus H minus square root of X , all divided by h . Again, I don't know what happens to this expression as H goes towards zero because I've got H in the denominator and that messes things up.

Instead, let's see what I can do here. When I've got square roots like this and I've got the difference between two square roots, I can use the idea of a conjugate to see if I can make some headway.

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Narrator: What I'm going to do is take the expression as written and then I'm going to multiply by the conjugate of the numerator. That's the same thing just with the plus sign. Then if I do the same thing in the denominator, this fraction is just one. I'm not changing the value of this thing, but maybe I can make a little bit of headway in simplifying it. Now, Now I've got an expression that I need to expand.

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Narrator: I've got square root of x plus H times another square root of x plus h That will just be X plus h Then I've got the square root of X times the square root of x plus H with a negative sign, but I've also got square root of x plus H times the square root of X with a positive sign. Those two will cancel out. That's the idea behind using conjugates is some of the partial products end up canceling each other. Then the only other piece I have to worry about is minus square root of X times plus square root of X . That's just minus X .

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Narrator: Then in the denominator, I've got H times this piece here. Square root of X plus H plus square root of X . Then now I see that I've got X minus X , those cancel and I'm left with an H in the numerator and an H in the denominator. The cancel too. I'm left with one in the numerator and square root of X plus H plus square root of X in the denominator.

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Narrator: Now, if I look at this and I ask myself what happens to this expression as H goes towards zero, then square root of X plus H is going to go towards square root of X . I've got another square root of X . This whole expression will tend towards $1/2$. Square root of X as H goes towards zero. Let's do a third example.

00:06:51:01 - 00:08:49:00

Narrator: I'm going to keep track of the examples here so we don't get lost so that we had first example and then second example and then third example is going to be one over X plus H minus one over X or over h , Here in my numerator, I've got two rational expressions, and I'm subtracting them. I need to think back to how to do that. How to do that is to use common denominators. I'll have one over X plus H times X over X minus and then X plus H over X plus H times one over X that whole thing is divided by h and so in the numerator, I've got an X minus an X minus an h all divided by X plus H times X , and that whole thing is divided by H , So in the numerator, I've got X minus X minus h so those X s are going to cancel. I'm going to have minus and then I've got H divided by.

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Narrator: This is in the denominator, the X plus H over X , and I've also got this in the denominator. I'll just wrote it like this. Now I can cancel the H s. And I'm left with minus one

over x plus H times X that as H goes towards zero, will become minus one over X squared. Let's think about a different example now.

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Narrator: Let's think about this expression. Three over X plus two minus three over A plus two and all divided by X minus A . In this example, I want to figure out what happens to this expression as the value of X goes towards the constant A . And unless I do something to this expression, I don't know what happens because if X goes towards A , the denominator goes towards zero. Again, this thing does not exist at zero when the denominator is zero.

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Narrator: I need to simplify this before I do that. This one's a little bit similar to the last one because I've got the difference between two rational expressions. In the numerator, I'm going to have to find a common denominator. Three and then X plus two times a plus two over a plus two and then subtract. That's two X plus two over X plus two, multiply that by three over a plus two that whole thing is divided by X minus a .

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Narrator: Next. I've got three A , and I've got six, and I've got minus three X , and I've got minus six. Then I've got X plus two times A plus two and the denominator and I've also got X minus A , so I'm just going to collect those together just to make this a little bit neater. I've got X minus A . I got X plus two and I've got a plus two.

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Narrator: The six is cancel, plus six and the minus six. I've got three A minus three X . I'm going to write that. I'm going to factor out the three, but I'm also going to factor out a negative sign because I notice down here, I've got X minus A . See how I've got minus three X and minus three times minus A , I've got plus three a .

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Narrator: I got X minus A and X plus two and A plus two. Now I can cancel the X minus A in the numerator and the denominator. I've ended up with minus three over X plus two, A plus two. And as X goes towards A , this is going to tend towards minus three over a plus two squared. Okay.

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Narrator: Let's do one last example with what's going to look like a horrendous expression initially and it is a little bit complicated, but these are the expressions that you might have to deal with moving forward. It's nice to get some practice in simplifying complicated expressions. Let's write this up here. We got four X squared. Then we got X minus three.

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Narrator: And I'm going to bring in a rational exponent, one half. Then I'm going to subtract X cubed. I'm going to throw in a constant of a half in there, and then I've got X minus three, and I'm going to throw in a negative a half for the exponent. And then I'll put in another constant of four and I'll divide that whole thing by X minus three, and we'll raise that to the

power one half, and then we'll raise the whole thing to the power three. Not a very pleasant expression to look at, but let's see if we can work with it and simplify it a little bit.

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Narrator: We have to start. I noticed that I've got X minus three in three different places in each piece. I've got some X s. I've got some constants in the denominator here. I know I've got an X minus three to the half and then cubed that's going to be X minus three to the $3/2$.

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Narrator: It's often a good idea whenever you see fractional exponents like this to try and get rid of them somehow. One way to do that is if I was to multiply this piece of the expression by X minus three to the half, then X minus three half times X minus three to the half would be just X minus three. So let's see if by doing that, we can make any progress. So essentially, what I'm going to do is multiply this whole thing. By X minus three over X minus three. To the half.

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Narrator: Just multiplying by one, so I'm not changing the value of this expression, but hopefully I can make some progress. If I think first about this first term, the X minus three to the halves will just collapse into X minus three. Then this next piece, I'm going to put the four here and the half here together. One half times four is two, and there's not a lot I can do with the X cubed yet, so I'll just pull the two out in front. And then I've got X minus three to the minus a half times X minus three to the plus a half.

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Narrator: That's just going to reduce down to the number one because I'm going to remember when we're multiplying numbers with exponents, we add the exponents, minus a half plus a half is zero, if we have an exponent of zero, then the number becomes just the number one. I don't even need to write it down because I'm just multiplying by one at this point. So it's already looking quite a bit better. In the denominator, I've got X minus three to the $3/2$ times X minus three to the one half. That's going to be X minus three to the $3/2$ plus one half, which is two.

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Narrator: And so I think all that's left is for us to clean up the numerator. I've got four x cubed minus two X cubed. I got two x cubed, and then I've got $-12 X$ squared. Then I can pull out a common factor here of two X squared and I'll have X minus six. A couple of things left here is whenever we're dealing with rational expressions, we have to think about the domain and the expression is not going to be defined if the denominator is zero.

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Narrator: But because we've got some fractional exponents here, we also have to make sure that what's inside a fractional exponent term has to be greater than zero. Because we have X minus three raised to the power one half here, we know that X is going to have to be bigger than three. X has to be bigger than three here. That takes care of the X is not going to

be able to be three either, but if X is bigger than three, then it can't be three. This is going to be the domain restriction.

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Narrator: We're good for all X greater than three. Then why do we even do this thing? Well, one reason we might do this thing is we're often looking to see when an expression like this is equal to zero, which effectively means the numerator being equal to zero. In this initial form that the expression was in, it's very hard to see where the numerator is equal to zero. But now that we've simplified it to this form, we can see right away, well, if X was six, then this expression would be zero.

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Narrator: That's called finding the root of an equation when we're trying to find when it's equal to zero. The root of this expression is x equal six. If we weren't paying attention, we might say, well, x equal zero is also a root, but X has to be bigger than three. X equal zero is irrelevant here. This expression is only defined for X bigger than three, the only root is x equal six.

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Narrator: So at this point, we've gone through five different examples, getting a little bit more complicated as we go along. I'm going to give you a couple of examples now to work through by yourself and pause the video, work through these examples and see if you can come up with the answers. For the first one, let's do X plus two over X minus A plus two over A and divide by X minus A . This is similar to example number four. So work with this expression and try and simplify it and get it to a different form in which we can figure out what expression is tending towards as X goes towards A .

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Narrator: Then for the other one, the second one is going to be a little bit more like this one, a little bit harrier. Let's three X plus two to the half, and then two X plus three minus one, and then constant two minus two X plus three. Then let's have a constant of one half, and then a three X plus two. And an exponent of minus a half and this whole thing divided by three X plus two to the one half. Okay. Let's see how you did. Let's do the first one.

00:24:16:99 - 00:25:23:52

Narrator: Looking at the numerator, we're taking the difference of two rational expressions, so we need to get common denominators going here. The common denominator will be XA . We'll have X plus two over X times over A minus A plus two over A times X over X , and all of that divided by X minus A . And so in my denominator, I'm going to have $X A$, and X minus A , and in my numerator, I'm going to have a times X , two times A . Minus a times X minus two times X .

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Narrator: Then in my denominator, I got a X and X minus a . I can see that the Ax minus Ax will cancel, and I've got $2a$ minus two X . Again, I'm going to do the same thing and I did in

the previous example, I'll pull out a factor of minus two so that I can write X minus A . Then we got a X , X minus A . Now I can cancel the X minus A 's.

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Narrator: My final answer is minus two over ax as X goes towards A , that's going to go this expression will go towards minus two over a squared. Next, the second one, there's a lot going on here. You've got to go slow and steady. One thing to notice is we've got a positive exponent here and the negative one here. Then similarly for this term here, we've got a positive exponent here of one that's hidden and then a negative exponent here.

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Narrator: The way to think about this is this is really a rational expression just written in a slightly different way. It's three X plus two to the one half, divided by two X plus three. Then we've got a two here. Let me put the two there. Then this piece, there's two X plus three.

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Narrator: Divided by three X plus two to the half. Then this half, we can think of this as having a two in the denominator piece. I'll put two there. And then that whole thing is divided by three X plus two to the half. I haven't really done any simplifying yet.

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Narrator: All I've done is change the way I've written this down. Because now that it's written like this, we see that we need to do common denominators to be able to subtract these two rational expressions. The common denominator is going to be two X plus three times two times three X plus two to the half. And because it's in the denominator, I'll combine it with the other expression that's also in the denominator, three x plus two to the half that three x plus two to the half here and here, we'll just combine and write down three x plus two. In the denominator, we're going to have two and then two X plus three and then three x plus two.

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Narrator: And next in the numerator now. I've got my two times three X plus two to the half and I have to multiply that by. Let's see. Another two and a three X plus two to the half. The two twos will become four and the three X plus two d a half times three X plus two to a half will become three X plus two.

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Narrator: Then I'm subtracting. Here I've got two X plus three and I need to multiply that by another two X plus three. Two X plus three squared. Okay. Next, let's simplify the numerator.

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Narrator: I think I can go right here. So we're going to have 12 X . And eight. Then I need to expand this. $2X$ times two X would be four X squared.

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Narrator: Then we've got two X times three, which should be six X and then three times two X , which would be another six X , we got $-12X$. Then finally, we got three times three, nine, so we've got minus nine. And then let's collect terms. Hey, look at that. The $12X$ s go away.

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Narrator: So we've got minus four x squared and then we've got plus eight minus nine, so we've got minus one. I want to write this as minus and then four x squared plus one. That takes care of the minus four x squared minus one. Then in the denominator, we've just got what we had before. Two $2X$ plus three and three X plus two.

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Narrator: Let's think about the domain because we always need to worry about the domain when we're dealing with rational expressions or complicated expressions like this where we've got things in the denominator and we want to also when we have fractional exponents. For this one, if I'm looking at this one half exponent here, I need three x plus 2 to be positive, which means X is going to have to be bigger than minus two thirds. So let's just write that here for all X bigger than minus two thirds. Let's just see if there's any other restrictions around there. X bigger than minus two thirds will be okay for this piece as well, and for this piece, and for this piece.

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Narrator: In terms of the denominator being potentially zero, as long as X is bigger than minus two thirds, we're okay. There's our domain and then why would we even do all this? As with this example, it's typically because we want to find out where the roots of the expression are. In other words, where does this expression equal zero? The numerator of this expression equaling zero, well, that's really hard to figure out.

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Narrator: We can easily look at this form of the expression though and see that the numerator of this expression will never be zero. Because X squared has to be greater than or equal to zero, the smallest it could be is zero, but we've got the plus one here, so we cannot make this expression zero. This expression has no roots. That's all for this video. In the next video, we'll start moving into a different area and start thinking about solving equations and we'll start with linear functions.

00:40:16:46 - 00:40:28:02

Narrator: In the next video, we'll change direction and start thinking about solving equations and in particular, we'll start with linear equations.