Solving Linear Equations

Transcript

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00:00:00:08 - 00:00:29:05
Narrator: Hello, and welcome to video number 16 in this series. In this video, I'll begin to review solving equations, starting with linear equations. I'll define a linear equation in one variable in the following way, AX plus B equals. Zero. Here, A and B are real numbers and A is not equal to zero.

00:00:29:05 - 00:00:58:74
Narrator: X is our variable here. What it means to solve this equation for X means I'm trying to find the value of X that will make both sides of this equation equal to one another. And another way of framing that is to say, I want to find the root of this equation. For an equation like this, there's only a single value of X that satisfies this equation. There's only one root.

00:00:58:74 - 00:01:36:08
Narrator: As an example, let's do two X minus six equals zero. So if I want to solve that equation, I'm wanting to isolate X on one side of the equation so that there's only numbers that we can evaluate on the other side of the equation. We talked about that in an earlier video, and we have two basic strategies for doing that. We can add a number to both sides of the equation. Or we can multiply both sides of the equation by a number.

00:01:36:08 - 00:02:10:50
Narrator: Let's employ both those strategies to solve this equation. First of all, remember, I'm trying to get X on one side and just numbers on the other side. I can see on the left hand side, I've got a minus six. If I add six to both sides of the equation, then that will cancel this minus six on the left hand side, and I'll be moving a number to the right hand side, but leaving that part of the expression involving X on the left hand side. Let's do that first.

00:02:14:95 - 00:03:14:57
Narrator: So now the minus six and the plus six cancel, and I've got two X equals six. Then now if I multiply both sides of the equation by a half. Now, the half times two will cancel to become one times X, so just X and where you have a solution, x equals a half times six or

three. Anytime you solve an equation like this, check your answer by taking your solution and plugging it back into your original equation. We're going to check that two times three minus six does indeed equal zero.

00:03:14:57 - 00:04:08:52
Narrator: Two times three is six, six minus six is zero. So a linear equation of this form only has one solution, as I say. In this example, it was X equal three, and we can see this if we draw a graph of what this function looks like. If I set up some axes, my horizontal axis represents X, and then the vertical axis represents this equation here. One way to think about this is to write the vertical axis is Y and then I'm going to graph the function or the equation Y equals to X minus six.

00:04:09:25 - 00:05:20:87
Narrator: So that's going to cross this vertical axis at minus six because when X is zero, Y is going to be minus six and it'll cross the X axis at the value we just found, X equal three, then we can draw a line passing through these two points, I straight line and that's going to represent the function Y equals two x minus six. Where this line crosses, the X axis, the horizontal axis is the solution to the equation that we started with two X minus six equal zero. We're finding which value of X results in a value of Y equals zero. The answer is an X equal three results in a Y of zero. Let's do a couple more examples.

00:05:22:79 - 00:06:09:98
Narrator: Let's solve three X minus four equals eight. The form of this example is a little bit different to the earlier form because I don't have zero on this side. But this is equivalent to if I wanted to make zero on this side, it's equivalent to if I subtract eight from both sides, I could have three X -12 equal zero. But I'm going to start with this expression instead. First thing I want to do is I want to get rid of the minus four, I'll add four to both sides.

00:06:09:98 - 00:06:38:43
Narrator: This time, I'm not going to write all the steps. I'm going to compress the amount that I'm writing just to save a little bit of time. If I add four to both sides, that will cancel on the left hand side and just add four to the right hand side. I'll have three X equals, and then eight plus four is 12. Then if I divide both sides by three, I have X equals 12/3 or four.

00:06:38:43 - 00:07:14:28
Narrator: Then again, I want to check that my answer works. I need to check that three times four minus four does equal 8 and 3 times four is 12 minus four is eight, so it does work out. Let's do one more example. Let's do an example involving fraction. Let's do -5/4 times X plus two equals minus three.

00:07:16:20 - 00:08:06:55
Narrator: I'll deal with the plus two first by subtracting two from both sides. I'll have minus three minus two on the right hand side, and then I'm going to multiply both sides by -4/5. Because -4/5 times -5/4 would be equal to one. I'll just have X on the left hand side, and I'm

just going to write this one out because sometimes fractions, it's easy to make mistakes. I got -4/5 times Times, I'll use a dot times five.

00:08:06:55 - 00:08:28:83
Narrator: And so here my fives again, I cancel, and I'll be left with X equal minus four, which is not right. What did I do? Oh, it's I got a minus. I do have a minus. I do have a minus.

00:08:30:11 - 00:09:10:85
Narrator: Okay. So we fine. So just B, rewind. And so we end up with the minus four, and again, just check that but it's correct. Plug in minus four into our original equation, -5/4 times minus four plus two, and we want that to equal minus three, -5/4 times minus four would be no.

00:09:10:85 - 00:09:20:88
Narrator: What am I doing? No, plus. Oh, my God. Yeah. No, I'm doing something wrong.

00:09:20:88 - 00:09:37:52
Narrator: So it is Oh, it's 'cause I got a minus here. Yep. Jesus. Jesus. Okay, so we got to go back to that part.

00:09:49:70 - 00:10:31:22
Narrator: Next, I need to isolate the X on the left hand side by multiplying both sides of this equation by -4/5. The -4/5 times -5/4 would become one. I've just got one times X on the left hand side, and then on the right hand side, I've got -4/5 times minus five. So the fives cancel. I just have to be careful with my negative signs.

00:10:31:22 - 00:11:28:84
Narrator: I've got negative four and then a negative here. That's going to become positive four. I then just need to check that works. I got -5/4 times four plus two, -5/4 times four is minus five minus five plus two is minus three, everything's good. Another way in which we can use linear equations like this and try and find a value of X that satisfies some linear equations is if we have two linear equations, and we're trying to find the value of X that makes the two equations equal to one another.

00:11:28:84 - 00:13:13:26
Narrator: For example, if I have the linear equation minus X plus two X minus three, and then minus three X plus seven. What I'm going to do actually is I'm going to call this one Y one and then this one Y two. I've got two equations here, and we could think of these as representing two lines on a graph. For example, this first line is going to cross the Y axis at y equals minus three, then it's going to it's going to have a positive slope because the slope is given by this number here, the plus two, and it's going to go up this way, this is Y one, equation the Y one equation, and then the Y two equation is going to be negatively sloped with a slope of minus three and it's going to cross at maybe I'll just make this go up a little higher at seven, w equal seven, it's going to come down like this. And what I'm trying to find is I'm trying to find that point there.

00:13:13:42 - 00:14:00:78
Narrator: Which value of X would make both of these linear equations equal to one another, while we're at it, we'll find the value of Y as well. We call that X zero and Y zero. To do that, all I do is set these equations equal to one another. Two X minus three equals minus three X plus seven. Then what I'm wanting to do is I'm wanting to have X isolated on one side of the equation and just numbers on the other side.

00:14:00:90 - 00:14:34:30
Narrator: First thing I could do here is to get the Xs all on this side, I could add three X to both sides. If I add three X to the right hand side, it's going to cancel the minus three X here. Then if I add three X to this side, I'll have five X. And over here, I won't have any xs. Then in terms of the numbers, the seven and the minus three, I want to move this minus three over here.

00:14:34:30 - 00:15:07:72
Narrator: If I add three, that would cancel it over here and add three to this side. I've already got seven here and now I've got seven plus three, which is ten. Then the last step, if I just divide both sides by five, I'm going to have X equal two. Let's check that we got the right answer. The way to check this is to plug in X equals two into both one equations and make sure we get the same value of Y.

00:15:08:75 - 00:16:06:67
Narrator: So let's do the first one, two times two minus three would be four minus three would be one, and then the other equation minus three times two plus seven, minus three times two is minus six plus seven would be one. This value on my graph is one and this one here is two. The two equations cross when X is two and Y is one. At this point, pause the video and I'm going to give you a couple of problems to work on and see if you can work through and get the correct answers. The first one is going to be more with this flavor.

00:16:06:67 - 00:16:47:38
Narrator: So let's solve this equation for X. So for X, and we're going to have minus two X plus three equals one. Find the value of X that makes both sides of this equation equal to each other. Then the other one, we're going to have two equations for X. It's going to be more like this example.

00:16:47:38 - 00:17:34:49
Narrator: We'll have let's see, 5/3 X minus nine. And we want that to be equal to minus two X plus two. Then remember once you've got your solutions to check them by plugging in the value that you obtain into the equations to make sure that they work. Let's go through the first example, first problem. We've got minus two X plus three equals one.

00:17:34:49 - 00:18:34:02
Narrator: I want to isolate X on one side of the equation, let's subtract three from both sides first. If we subtract three here, it will cancel with the plus three and we've got one minus three, so we'll have minus two. Then if we divide both sides by minus two, we have X on the

left hand side and on the right hand side, minus two divided by minus two is one. Then let's check. So we'll plug in the value X equals one into our equation and minus two times one is minus two minus two plus three is plus one, which is what we were hoping it would be, that's okay.

00:18:34:02 - 00:19:07:87
Narrator: Then the second problem, we got five thirds of X minus nine. Equals minus two X plus two. I'll try and get all the Xs on the left hand side. We'll add two X to both sides. We'll have five thirds, X plus two X.

00:19:08:15 - 00:19:43:72
Narrator: That will get rid of the X on this side, and then to get rid of the minus nine on this side, we'll add nine to both sides of the equation. We'll have two plus nine. Now I need to add five thirds and two. A common denominator there would be thirds, so two, if I want that to be in thirds, I need six thirds would be two. I got six thirds here and five thirds here.

00:19:43:72 - 00:20:22:93
Narrator: I got 11 thirds of X. Over on this side, I've got 11. Then if I multiply both sides by 3/11, then we'll end up with X equal three and checking that that's the right answer. So in my first equation, 5/3 times three minus nine. 5/3 times three is five, five minus nine is minus four.

00:20:22:93 - 00:20:53:47
Narrator: My other equation minus two times three plus two minus two times three is minus six and plus two is minus four. They're the same, so that checks out as well. That's it for this video. In the next video, we're going to turn our attention to quadratic equations and look at solving those equations by factoring.