Solving Linear Inequalities

Transcript

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Narrator: Hello, and welcome to video number 19 in this series. And in this video, I'll review inequalities, starting with linear inequalities. So in an earlier video, we talked about representing inequalities such as X greater than four as representing all real numbers that are greater than the number four. And we could think of this on a number line if we have zero on a number line here, and then one, two, three, four, then this inequality represents everything on this side of the number four, not including the number four. So the round bracket here indicates that we're not including the number four.

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Narrator: And we can also represent this using interval notation as X belongs to the set of real numbers that are in the interval between four and infinity. And we're not including the number four, so we've got a round bracket here, and we're not including the number infinity, so we've got a round bracket here as well. Other examples of inequalities would be X greater than or equal to four. So we would write that using interval notation with a square bracket. And just a square bracket on the left hand side for the four because we're including four, but we're still not including infinity.

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Narrator: And then we could also go with a less than. So that would be X in the interval between minus infinity and four, not including both endpoints. And then we could also have X less than or equal to four. So that's going to be with a round bracket on the left side, and then a squared off one on the right side. And then we can also have what's called a continued inequality.

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Narrator: So we could have X in between, let's say, minus four and plus four. And so this would be represented using interval notation as X belongs to the interval between minus four and plus four. So next, we want to move into solving inequalities. And that means, if we have an inequality like the following, so seven X minus four less than or equal to four X plus

five, we want to find all values of X that satisfy this inequality here. And so it's similar to solving an equation, except instead of an equal sign, we have an inequality, but we employ the same methods, so we can add something to both sides or we can multiply both sides by a number.

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Narrator: So let me show you how that works in this example. So the idea, just as with solving an equation is to get X onto one side of the inequality and just numbers on the other side. So let's have the Xs go onto the left hand side. So if I subtracted four X from both sides, then that would cancel the four X on this side. So we'd have seven X minus four x.

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Narrator: And then for the numbers, we want to get rid of the minus four on this side. So if we add four to both sides, then the minus four and the plus four would cancel on this side. So now we have three X less than or equal to nine. And then finally, we can just divide both sides by three. Or we could think of that as multiplying both sites by one third, same thing, and we get X less than or equal to three.

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Narrator: And we would represent that using interval notation as X belongs to the interval between minus infinity and three, we're including the value three. Just as when we're solving equations, when we're solving inequalities, it's a good idea to check that we've got the right answer. One way to check is to pick a number that's in the interval that we ended up with. So in this case, if we picked something less than or equal to three, let's pick three. Then we could plug in three on both sides and make sure that the left hand side is less than or equal to the right hand side.

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Narrator: So in this case, we'd have seven times three is 21 minus four, so it's 17. And on this side, we've got three times four is 12 plus five is 17. And so 17 satisfies this inequality because we've got 17 less than or equal to 17, which is a true statement. We have to be a little bit careful when we're multiplying both sides of the equation by a negative number, because what happens there is the inequality reverses direction. And we know that this is the case because, for example, if I say one is less than two, which is true, if I multiply both sides of this by minus one, we'd have minus one and minus two, but minus one is not less than minus two.

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Narrator: Minus one is greater than minus two. So anytime you multiply both sides of an inequality by a negative number, we have to change the direction of the inequality. So for an example of how that might come into play in solving an inequality like this, let's do three X plus two. And let's do less than five X plus six. And we want to solve this inequality, which means isolating X on one side, just have numbers on the other side.

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Narrator: We're trying to find the solution set of Xs that would make this inequality a true statement. So let's move the five X over onto the left hand side by subtracting five X. And then let's move the two on the left hand side to the right hand side, by subtracting two. So now we've got minus two X less than four. And so now to isolate the X on this side, we're going to multiply by minus a half.

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Narrator: And so if we multiply by minus a half, we're going to have X on left hand side, and four times minus a half would be minus two. But because we're multiplying both sides by a negative number, we have to remember to change the direction of this inequality. So now that becomes greater than. So the solution said in this case is X in the interval from minus two up to infinity. And we could check if we pick a number greater than minus two, let's say minus one.

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Narrator: Let's go back to our original inequality to make sure that we've done it correctly. If we plug in minus one here, we would have three times minus one is minus three plus two, we'd have minus one here and five times minus one minus five plus six. We'd have plus one here, minus one is less than plus one, it looks like we've done this correctly. So we talked about a continued inequality here where we had X in between minus four and plus four. We can also solve inequalities of that nature.

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Narrator: So, for example, let's switch to a different color. Let's do minus one, less than minus four X plus 3/5, and let's have less than or equal to three. So we want to find all values of X that would satisfy this continued inequality, and we solve it in a similar way to the way we solved the inequalities we looked at earlier, but we just have to remember that we've got kind of three pieces to worry about. So whatever we do to one piece, we have to do to the other two pieces. So, for example, the first thing I could do here, I'm going to try and isolate X in the middle.

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Narrator: So first of all, I want to get rid of that divided by five. So I'll just multiply each piece by five. So I'll have minus five, less than, minus four X plus three, less than or equal to three times five, 15. And then let's get rid of the plus three. So I'll subtract three from all three pieces.

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Narrator: So we'd have minus eight, less than minus four X, less than or equal to 12. And then finally, let's divide by minus four. So I'm dividing by a negative number, or think of it as multiplying by minus a quarter, multiplying by a negative number. So I have to reverse the inequality signs. So I'm dividing by minus four or multiplying by minus a quarter.

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Narrator: So this is going to become two and then X in the middle. And then this one, dividing by minus four. So that will become minus three. And so typically the way you write a continued inequality is to have smaller numbers on the left and larger numbers on the right. So this is kind of written in reverse.

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Narrator: So I'm just going to rewrite this as minus three, less than or equal to X, less than two. And in interval notation, it would be X is in the interval between minus three and two. Now we're including minus three, so we've got a square bracket on that side, and we're not including two, so we've got a round bracket on that side. And let's just check. Let's pick a number inside this interval, say zero and see if that satisfies the inequality we started with.

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Narrator: If we had zero plugged into here, we'd get three fifths, and three fifths is in between minus one and three. So it looks like we've done it correctly. So at this point, pause the video and see if you can work through the next two problems that I'll put up and see if you can solve those inequalities. So that means find all values of X that satisfy the next two inequalities. So the first one will be nine x plus two greater than or equal to five X minus two.

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Narrator: And then the second one, let's do minus five, less than or equal two, minus two X -5/3, less than one. Okay, so let's see how you did. So for this first one, we're going to try and move the Xs to the left side numbers to the right side. So first thing we can do is subtract five X from both sides. So nine X minus five X would be four x.

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Narrator: And then we'll try and move the two over onto the other side by subtracting two. So minus two minus two be minus four, and then we'll divide by four. So we'll get X greater than or equal to -4/4 will be minus one. And in interval notation, that would be X is in the interval from one, sorry, minus one up to infinity. And we could check.

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Narrator: Let's pick a number in this interval to check with. Let's pick, say, the number one. Nine times one is nine plus two would be 11, five times one minus two would be three. So yeah, 11 is greater than requ to three. So that's a check that we've done this correctly.

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Narrator: Doesn't mean for sure we've done it correctly because it's just one value, but at least it gives us some reassurance that we haven't made a silly mistake. For number two, first thing, multiply all three pieces by three, -15, less than or equal to minus two X minus five, less than three. And then next, let's add five to all three pieces. So -15 plus five would be minus ten, less than or equal to minus two X, less than three plus five is eight. And then we'll divide by minus two.

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Narrator: And remember, if we're multiplying or dividing by a negative number, we have to switch the directions of the inequality signs. So I'll just do that first so I don't forget. And then I'll divide through by minus two. So we'll have X left in the middle and minus ten divided by minus two, it'll be five and eight divided by minus two, it'll be minus four. And then I'll just write this in the usual fashion.

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Narrator: So with the smallest number on the left, the biggest number on the right. So minus four less than X less than equal less than or equal to five. And so in interval notation, that would be X is in the interval between minus four, and we're not including minus four, so we've got a round bracket there, and then plus five. So we've got a squared off bracket there. And last step is just check with a number in this interval, say zero is inside this interval.

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Narrator: So let's plug in zero here. We'll have minus five thirds and minus five thirds is in between minus five and plus one. So that provides some reassurance that we've done this correctly. So that's it for this video. And in the next video, which is the last video in this series, I'll stay with inequalities, but I'll look at inequalities that involve polynomials, rational expressions, and absolute values.