Solving Other Inequalities

Transcript

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00:00:00:00 - 00:00:50:29
Narrator: Hello, and welcome to the final video in this series, number 20. In this video, I'll review some examples of solving inequalities that involve polynomials and rational expressions and absolute values. In the last video, we reviewed linear inequalities like this, seven X minus four, less than or equal to four X plus five, we solved this inequality and ended up with the solution that X less than or equal to three, which we can also write in the form. X is in the set. In the interval between minus infinity and plus three, not including minus infinity, but including three.

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Narrator: But what if we have a inequality that involves some squared terms? For example, X squared plus four X minus three, less than or equal to five X plus nine. So we want to solve this inequality, which means finding an interval or multiple intervals for X that satisfy this inequality such that if we plug in any value of X in those intervals, the left hand side would be less than or equal to the right hand side. So we employ a similar method when we're solving this inequality, which in this case is quadratic. But you'll see that the end result is a little bit different.

00:01:45:95 - 00:02:26:01
Narrator: In this case, what I want to do is I want to move everything onto one side of the inequality and just leave zero on the other side. So similar approach to the way we solve linear inequalities, but slightly different because I'm trying to get zero on one side. So all I need to do is I need to subtract five X and subtract nine from both sides. I have X squared and four X minus three, and then minus five X minus nine. Now I've got zero on one side, and now I just need to collect terms here.

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Narrator: Only got one X squared term. I've got a five X minus four x, so that's going to be minus X, and then I've got a minus three minus nine. That's going to be -12. Then our

instinct ought to be whenever we see a quadratic like this, can we factor it? To numbers multiplied to get -12 and add to get minus one.

00:02:55:41 - 00:04:03:38
Narrator: Four and three, one of them plus, one of them minus should do that. In this case, we want a minus one, we'll need minus four and plus three. Now that we've expressed the inequality in this form, and we know that this quadratic function represents a parabola that opens upwards, so it's more of a valley than a hill because there's a positive coefficient on the X squared plus one. So we know that the roots of this parabola are at four and minus three. That means the Parabola crosses the horizontal axis at minus three and plus four, we want to know where this quadratic function or quadratic equation is less than or equal to zero.

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Narrator: In other words, we're interested in this part of the graph because that's where it's less than or equal to zero. In other words, we've discovered that the solution set here is X in between. Let's do it like this. X is going to be in between minus three and four. This is less than or equal to sines because we've got a less than or equal to sign here.

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Narrator: It includes the two roots of the equation minus three and four. In interval notation, we say that X is in the interval between minus three and four. As usual, whenever we solve an inequality like this, it's good to see if at least if we plug in a test value that it does give the right result. So our test value could be let's actually do three test values here. Let's do one on the left of this interval, say minus four.

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Narrator: So X equal minus four, and then we'll do one inside the interval. Let's say do X equal one. That's a number inside this interval, and then there's do one on the right hand side. Let's do X equal five, see what happens to the inequality that we started with, which is up here. So if I plug in minus four on the left hand side, I'm going to have 16 minus -16 minus three, and we want that to be less than or equal to five times minus four B -20 plus nine.

00:06:16:37 - 00:06:53:50
Narrator: We're going to have minus three. On the left hand side, we're going to have -11 on the right hand side, minus three is not less than -11. This statement is false. And for the value in between the one, we're going to have let's see one, write it down, one and four minus three. We want that to be less than or equal to five plus nine.

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Narrator: And on this side, we've got five minus, we got two and on this side, we've got 14. That's true. We've got a true statement there, and then on the right side of the interval, if we pick five, we have 25 and 20 minus three. We want that to be less than equal to 25 and nine. Let's see.

00:07:26:12 - 00:08:10:49
Narrator: We got 25 and 20 so 45 -42 on this side, and on this side, we've got 34, so that's false. This checks out because we wanted to find the values that satisfied the inequality and we ended up with everything in between minus three and four. So when we picked a value in between minus three and four, everything checked out. When we pick two values that were not inside this interval, then the inequality did not work out. This is a nice way to check a quadratic inequality once you've solved it.

00:08:10:49 - 00:08:54:28
Narrator: Pick a couple of values outside the interval that you ended up with and see if the inequality doesn't work out, and then pick a value inside the interval that you ended up with and see if it does check out. That's an example of a quadratic inequality. Let's look at inequality involving a rational expression. Let's do X plus two and X minus one. Let's see where this rational expression is greater than a equal to zero.

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Narrator: The approach with rational expressions is a little bit different. It's more of just thinking things through. First thing that I pay attention to when I'm looking at a rational expression is where it's not defined. If X was one, we would be trying to divide by zero and we're not permitted to do that. X equal one, this is undefined.

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Narrator: Another interesting value is equal minus two. Because then this rational function or rational expression is equal to zero. We're trying to find when the expression is greater than the equal to zero. We've found one solution, x equal minus two. So we've got two key values here.

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Narrator: The next step is to look at some test values that are in the intervals defined by these key values. What I'm thinking about is if I've got a number line and I've got the value minus two and the value one, then what I want to do for my test values is pick something on the left of minus two, say, minus two minus three. It's to the left of Sorry. No, that's right. What we want to do is pick values in each of the intervals defined by this number line.

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Narrator: To the left of minus two. For example, minus three, and then something in between minus two and one. So zero. And then something to the right of plus one, say, two, and see what the sign of the function is. So if we have X equal minus three, then we would have minus one in the numerator and we'd have minus four in the denominator, and that is a negative divided by a negative, that's greater than zero.

00:11:49:05 - 00:12:33:85
Narrator: If we have zero in place of X, we would have two plus two in the numerator and minus one in the denominator, that's going to be negative. Then if we have X equal two, we would have plus four. In the numerator and we'd have one in the denominator and that's a

positive divided by a positive, that's going to be positive. We're looking for where this rational expression is positive or zero. The only place where it is zero is actually called minus two and it is positive here and here.

00:12:34:89 - 00:13:59:60
Narrator: Putting all these pieces together, but we want to end up with a statement like this one. For which this rational function is, in this case, greater than equal to zero, and we've discovered that it works when we are on the left hand side of minus two, including minus two, it doesn't work in between and it works on the right hand side, but not including the value one, because at the value one, the function is undefined. So X is either less than or equal to minus two or X is bigger than one. In interval notation, the way we write that is X belongs to the interval between minus infinity and minus two, including the value minus two, In set notation, we denote or using the union symbol here. Or the interval between one and infinity, and we're not including the value one.

00:14:01:72 - 00:14:37:35
Narrator: Dealing with rational expressions, it's steps. You just go through the steps, thinking about key values and thinking about test values defined by the intervals that are set up by those key values. L et's move to an example involving absolute values. Let's do 2X minus three, absolute value. Less than five.

00:14:37:35 - 00:15:10:75
Narrator: We want to find all values of X satisfy that inequality. If we think about what this means, it means that two X minus three as a number must be in between. Minus five and plus five. This continued inequality is another way of writing this inequality here with the absolute value. Now, we've already seen how to solve this example.

00:15:10:75 - 00:15:59:21
Narrator: We did an example like this in the last video, but I'll just run through the steps real quick. Let's add three, first of all, minus five plus three would be minus two and then five plus three would be eight, and then divide through by two, have minus one, less than X, less than four. In other words, X is in the interval between minus one and four. Now we could check, say with the value one. Two times one is two minus three, minus one, take the absolute value minus one, we get plus one and plus one is less than five.

00:15:59:21 - 00:16:42:75
Narrator: We've got some reassurance that we've done this correctly. Let's do another type of example with an absolute value in it. Let's do four X minus six, greater than two. Thinking about what this means, it means that the number represented by four X minus two is either less than minus two or four X minus six is greater than plus two. Because then when we take the absolute value of it, the absolute value would be greater than two if either of these conditions holds.

00:16:43:96 - 00:17:04:12
Narrator: We have two inequalities to work with two linear inequalities. So we just have to work both of them. This one first involving the negative two. Let's add six to both sides. Minus two plus six would be four.

00:17:04:12 - 00:17:59:32
Narrator: This one is X less than one, and then the other one is to add six to both sides. We've got four X greater than two plus six would be eight. X greater than two. And in interval notation that would be X is in the interval between minus infinity and one, we use our union notation or the interval between two and plus infinity. Again, we could check with numbers in either one of these intervals.

00:17:59:49 - 00:18:36:33
Narrator: And check that our original inequality works. For example, something that's in this interval here, say zero is in this one. So we would have the absolute value of minus six, which is plus six and six is greater than two, that works out. In this interval, we could pick the number three, four times three is 12, 12 minus six is six, absolute value of six is six and six is bigger than two. Both of those numbers work out.

00:18:37:90 - 00:19:36:05
Narrator: At this point in the video, I'm going to put a couple more problems up and pause the video and work through those two inequalities that I'll put up in a second and see if you can solve them. In other words, find values of X that will satisfy the following two inequalities. The first one, let's do a quadratic, like this one. Let's do five X squared plus two X minus five greater than X squared plus two X plus four. Then let's do an example involving absolute value.

00:19:36:05 - 00:20:33:90
Narrator: Let's do absolute value of minus three X plus six, and let's have that B less than or equal to nine. Let's see how you did on the quadratic inequality first. I want to move all the terms over to one side and leave zero on the other side. I'll have five X minus X squared, so I'll have four X squared and then I'll have two X minus two X, the X term actually goes away and then we'll have minus five minus four minus nine. Hopefully, when you got to this point, you recognized that this was a difference of squares, that should make you think about conjugates.

00:20:33:90 - 00:21:38:57
Narrator: We're going to have two X plus three times two X minus three. If we multiply this out, we get four X squared minus nine and then six X minus six X, so that cancels. X the roots of this are at X, so the roots are at X equals -3/2 and plus 3/2. The only thing that's left for us to do is just to think about what this would look like if we draw it on a graph as a parabola. We've got a positive coefficient on the X squared, so that means it's going to be opening upwards rather than opening downwards and more like a valley than a hill.

00:21:38:57 - 00:22:09:49
Narrator: The roots are at -3/2 and plus 3/2. So here and here. We're trying to find where this quadratic is greater than zero. The part of the function that we're interested in is actually this piece and this piece, which is different to what we did before where we looked at in between. In this one, we're looking at outside.

00:22:09:63 - 00:23:07:00
Narrator: So if I write down my intervals, and I've got a strict inequality here, it says greater than zero, not greater than or equal to zero. These two roots are not going to be included in my solution set. We want X less than -3/2 or X bigger than plus 3/2. In interval notation, that's going to be X is in the interval between minus infinity and -3/2, Union 3/2 to plus infinity. That's the solution for the quadratic example and then let's do the solution for the absolute value example.

00:23:07:00 - 00:23:57:81
Narrator: This absolute value inequality means that this value minus three X plus six is in between minus nine and plus nine. Then if I subtract six from all three pieces, I'm going to have -15 on this side, minus three X in the middle and I'm subtracting six, so I'll have three on that side. Then I'll divide all through by minus three, I'll have minus five. Be I'm divided by a negative number, I have to switch the direction of the inequalities. Sorry I'm dividing by minus three, so I'll get plus five.

00:23:57:81 - 00:24:38:92
Narrator: X and then three divided by minus three, minus one. Then I'll just switch this around so it's in the conventional direction. X in between minus one and five, including both endpoints. In interval notation, that would be X is in the interval between minus one and plus five. That's it for this video and this being the last video, that's it for the entire video series.

00:24:38:92 - 00:24:42:08
Narrator: I hope you found these videos useful.