



Solving Quadratic Equations Using the Quadratic Formula

Transcript

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Narrator: Hello, and welcome to video number 18 in this series. And in this video, I'll review using the quadratic formula to solve quadratic equations in situations where factoring either isn't possible or it isn't particularly easy. So remember we have our definition of a quadratic equation, $Ax^2 + Bx + C = 0$, where A , B and C are real numbers, and A is not equal to zero. And the quadratic formula is a formula for solving this equation. So finding the roots.

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Narrator: And it's given by this formula. So it starts with a minus B , and then this symbol here plus minus means it's going to give us potentially two solutions, one where we use plus and one where we use minus. And then we have the square root of $B^2 - 4AC$, and then that whole thing is divided by two a . So it's kind of an ugly formula, but it's an effective one, and it works to solve this quadratic equation. So let's run through a few examples to see how to use it in practice.

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Narrator: So the first example I'm going to do is $x^2 + x - 6 = 0$. So we have to when we're using the quadratic formula, we have to know what A , B and C is. So A , in this case, is one, B is one, and C is minus six. So x will be equal to minus B , so minus one plus or minus square root, and then we have B^2 , so one squared, minus four, and then A is one, and then C is minus six. And then the whole thing gets divided by two times A , two times one.

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Narrator: So we got minus one plus minus. So this square root $B^2 - 4AC$, this thing is called the discriminant, the $B^2 - 4AC$, and that can either be positive, negative or zero. If it's positive, then we end up with two solutions. If it's zero, we

end up with one solution, and if it's negative, we end up with no real solutions. So this $B^2 - 4AC$ plays a critical role in how many solutions there are.

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Narrator: So let's see whether we have a positive number here, a negative number or zero, and that will tell us how many solutions we have. So in this case, we've got one minus four times one times minus six, four times one times minus six is -24 , but we're subtracting -24 . So that's the same as adding 24, one plus 24 is 25. We've actually got a number here that's positive, so we've got two solutions, but it's also a perfect square, so we can take the square root of it to get five, and then we're going to divide that by two. We've got two solutions, one where we use the positive, one where we use the negative.

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Narrator: The one with a positive would be minus one plus five is going to be $4/2$, will be two, and then the other solution minus one minus five, $-6/2$, will be minus three. So this example, this was actually an example we looked at in the last video, and we factored it and I factored into X plus three X minus two. So we could have done this example just by factoring and seeing that the two solutions are X equals minus three and X equal plus two. But for illustration, I wanted to show you how the quadratic formula works in an example that we actually already know what the answers are. So this confirms our factoring method.

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Narrator: Let's do an example where we only have one solution. So in this example, the discriminant is going to turn out to be zero. So we'll do four X squared, $-12X$ plus nine equals zero. So let's use our quadratic formula. So we're going to have minus A , sorry, minus B .

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Narrator: B is -12 , so minus -12 is plus 12 plus minus square root of 12 squared. The fact that it's negative doesn't really matter when you're squaring it. So I'm just to save a little bit of writing, I'm just going to write 12 squared instead of -12 squared and amounts to the same thing. Minus four, A , C . Then the whole thing divided by two A , which is 8 So we got 12 plus or minus.

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Narrator: We got 144, and then we got minus four times four times nine. So we got -16 times nine, 16 times nine is 144. So we got $144 - 144$. So we've got plus or minus zero, and then divided by eight. So that's going to be $3/2$.

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Narrator: And I think that we did this example in the last video as well. Or if we didn't, we can see that it must factor because we're getting a nice rational number here. And it factors into two x minus three squared. So X equals $3/2$ is the single solution, just one solution here. Let's do a third example, one in which the discriminant is going to be negative and we have no real solutions.

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Narrator: So we'll do $X^2 + 4X + 5 = 0$. So X is going to be using the quadratic formula minus B , so minus four plus or minus square root of $B^2 - 4AC$. All divided by two times A . So we're going to have minus four plus or minus square root of, let's see, $16 - 20$, so minus four.

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Narrator: So we're going to try and take the square root of a negative number, which we can't do when we're restricted to real numbers. So no solution for this one. No roots. So these examples illustrate how the discriminant plays a key role here. If it's positive, we end up with two roots.

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Narrator: If it's zero, we end up with one root, and if it's negative, we end up with no roots. And the two examples where we got solutions, we could have factored and saved ourselves having to use the quadratic formula. So the quadratic formula is most useful when we can't do this, when we can't factor. Or when factoring is really challenging. So let's do an example where we can't do the factoring and see what the result of using the quadratic formula in that kind of example is.

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Narrator: So this is going to be example number four. So we're going to do $4x^2 - 8x + 1 = 0$. So using the quadratic formula, we start with minus B . So B is minus eight, so minus minus eight B plus eight plus or minus square root of $B^2 - 4AC$. So $B^2 - 4AC$.

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Narrator: B is minus eight, but minus eight squared is the same as eight squared, so just save some writing off just write eight squared and then minus four, A is four, and C is one. And divided by two times A would be eight. So we got eight plus or $-64 - 16$, which comes to 48. Divided by eight. I know we've got two solutions here because I've got a positive discriminant.

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Narrator: Let's see if I can clean this up a little bit. Let's do eight plus or minus square root of 48, if I factor that into a perfect square and another factor, then I could pull out the perfect square piece. I could pull that out of the square root. What I mean is 48 is also equal to 16 times three.

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Narrator: So now I can take the square root of 16. I could pull that out of the square root sign as a four because the square root of 16 is four. So now I'm going to have eight plus or minus four times the square root of three. And now I'm going to simplify this just by taking out the common factor of four here, because I'm dividing by eight. So I've got $8/8$, so that's one.

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Narrator: And then I'm plus or minus four eighths of the square root of three. So otherwise in other words, I could write it as the square root of $3/2$. So this is an exact way to write the two roots, one plus square root of $3/2$ and one minus square root of $3/2$. If we want to express this as a decimal, we're going to have to go to our calculator and it turns out that one minus the square root of $3/2$ is naught 0.13, four, and one plus the square root of $3/2$ is 1.866.

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Narrator: So this is an example where factoring would have been a challenge because we've ended up with an irrational pair of answers or a pair of irrational answers. If we wanted to draw what this quadratic equation looks like, sorry, this one, We know it's going to be a parabola that is more like a valley than a hill because it's got a positive coefficient on the X squared. It's going to pass through these two points on the horizontal axis. So if this is the X axis, this is the Y axis, zero is in the middle and then one and two. It's going to cross the axis somewhere here and somewhere here.

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Narrator: And it's actually also going to reach a minimum at the coordinate of x equal one, white equals minus three. So here. So it's gonna kind of look something like this. It's gonna go down. And then up. Oops. It's gonna

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Narrator: be more like this. Kind of like that. So this to another example where either factoring is not possible or it's just really challenging. So let's do this example. So, $24x^2 + 22x - 35$ equals zero.

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Narrator: So it turns out that this is possible to factor, but it's hard to imagine where to start with this. So instead, we're going to use the quadratic formula and see where we end up. So using the quadratic formula, so we start with minus B, so -22 plus or minus square root of B squared, so 22 squared, minus four, A, and then C, and then we're going to divide by two A, so divide by 48. So this is what we get when we plug in our numbers into the quadratic formula and so I can start writing down what I've got. So I've got -22 plus or minus, and then I've got the square root, and now we're starting to get into some big numbers.

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Narrator: So all the examples up to this point in the videos, we've been able to do just using knowledge of timestables and mental arithmetic. At this point, this is starting to get into territory where you're probably going to have to go to a calculator, which it's fine. So that kind of illustrates one drawback to the quadratic formula because often we do end up, especially in this discriminant part of the quadratic formula is you're working with often fairly big numbers and then you're having to take the square root. So you're going to have to

start using a calculator when you're applying the quadratic formula. And so that illustrates how if you can factor that's definitely the way to go.

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Narrator: So when you see a quadratic equation, your first instinct should be, can I factor this? And if after a few minutes you are struggling to find nice clean, easy factors, then go to the quadratic formula. Your instinct should not be to go immediately to the quadratic formula and your calculator. So at this point, we can't really factor this very easily. So going to a calculator to figure out what this comes to is pretty much the only thing we can do.

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Narrator: So if you do calculate this out using a calculator, you end up with 3,844 as the discriminant. So we know we're going to have two solutions because this is positive. And then if we take the square root of this number, turns out this is actually a perfect square, so we can take the square root. It comes to 62. And then All of these numbers are even, so we can pull out a factor of two, and we'll have -11 plus or -31/24.

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Narrator: And so if we do -11 -31, so that's going to be -42, and then -42/24, we can reduce that to -7/4 and then -11 plus 31. So that's going to be 20/24, so we can reduce that to 5/6. So those are our two roots for this example. So given that we've got rational numbers here, it turns out that we could have factored this and the factors we'll go this way. We end up with four X plus seven times six X minus five.

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Narrator: So this is what gives rise to the root -7/4, and this factor is what gives rise to the root 5/6. So, pause the video at this point and work on the next two problems that I'll put up and see if you can use the quadratic formula successfully to find the roots of the quadratic equations that I'll put up here. See if there's two solutions, one solution, or no solutions. So the first one, let's do two X squared plus three X minus one equals zero. And then the second one, let's do ten X squared, -19 X plus six equals zero.

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Narrator: Okay, let's see how you did with these two. So X equals minus B, so minus three plus or minus square root, B squared, so three squared, minus four, A is two, C is minus one. All divided by two times A is four. So I'll have minus three plus or minus. I'll have nine minus four times two times minus one is minus eight, minus minus eight would be plus eight.

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Narrator: Nine plus eight is 17. Okay. And if we write this as decimal numbers, so we'll have minus three minus root 17 all divided by four. That gives us -1.781 and minus three plus root 17/4.

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Narrator: That gives us 0.281. And then the second example. So we're going to have X equals minus B , be 19 plus or minus square root, B squared, so 19 squared, minus four, A is ten, and C is six, all divided by to a 20 . So we got 19 plus or minus.

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Narrator: And then if we evaluate this using a calculator, 19 squared minus four times ten times six, we get 121 . And that's nice because 121 is a perfect square, square root of 121 is 11 . So we've got 19 plus or $-11/20$. So $19 - 11$ will be eight. So $8/20$.

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Narrator: If we reduce that, we get $2/5$, and the other root 19 plus 11 would be $30/20$, so $3/2$. So those are our two roots, two fifths and $3/2$. And given that we've got rational solutions here, that implies that we could have factored this. So let's see what the factors are. So it must be um, one factor must be five X minus two.

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Narrator: That would give us the two fifths, and then the other one must be two X minus three. That gives us the $3/2$. And if we just check, this provides a useful check that we've done our quadratic formula correctly. We're going to have five X times two X , that'll give us a ten X squared. We've got minus two times minus three, that would give us the plus six, and then we got minus four X .

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Narrator: $-15 X$, that gives us the $-19 X$. So that's a nice check that we've done this correctly. So that's all for this video. And we've got two videos left in this series, and we're going to cover inequalities in those last two videos. So something a little bit different, and we'll start with linear inequalities in the next video.