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Solving Quadratic Equations by Factoring

Transcript

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00:00:00:00 - 00:00:51:82
Narrator: Hello, and welcome to video number 17 in this series. In this video, I'll review some examples of solving quadratic equations using the idea of factoring. So first, I'm going to define a quadratic equation as Ax squared plus Bx plus C equals zero. Here, A, B and C are real numbers and A is not equal to zero, what it means to solve this equation for X or to find the roots of this equation is we want to find any values of X so that the left hand side and the right hand side of this equation are equal. There's a number of different ways that we can do this.

00:00:52:18 - 00:01:56:14
Narrator: If we think back to some of the ideas that we explored earlier, it turns out that one useful way of doing this is using factoring. Back in, I think it was Video two, we talked about something called the zero factor theorem, and that said that if we have two numbers that multiply together to give us zero, then either A is zero or B is zero, or they're both zero. And in video 11, I think it was, we looked at factoring quadratic equations. It turns out if we put these two ideas together, we can solve this quadratic equation problem. As an example, let's work with X squared plus X minus six equals zero.

00:01:56:42 - 00:02:39:80
Narrator: And we can look at this and see if we can factor it. That should be our instinct. Anytime we see a quadratic equation, we should think about factoring it if we can. We need two numbers that multiply to give us minus six and add to give us plus one. In this case, if we have let's see, three and two and we want plus three and minus two because three times minus two will be minus six and plus three X minus two X will be plus one X.

00:02:39:80 - 00:03:14:06
Narrator: So if we reflect back to our zero factor theorem, we can see that we've got two factors that multiply together to give us zero. Either this is zero or this is zero. For this to be zero, X would be minus three. For this to be zero, X would be plus two. Those are the two roots of this quadratic equation, x equal minus three and x equal plus two.

00:03:14:69 - 00:03:46:17
Narrator: Can think of this in terms of a graph as well. Thinking back to the previous video where we looked at representing a linear function as a straight line on a graph, a quadratic function looks like a parabola, which is a curved function. If we think about these axis here, this is the X axis and this is the Y axis. We've got zero in the middle. This is often called the origin.

00:03:46:17 - 00:04:35:40
Narrator: Then if I mark off, one, two, if I mark off, this is two and then this is minus three. Because our parabola is going to pass through the point X equal minus three and Y equal zero here, and X equals two and Y was zero, which is here. Because the coefficient on X squared is a plus one, it's positive. It means that our parabola is going to be upward facing. It's going to be a curve like this and it's going to pass through these two points.

00:04:35:40 - 00:05:18:79
Narrator: The other point that it'll pass through is if X is zero, then Y would be minus six? It's going to pass through the point down here as well at minus six. The parabola is going to look a little bit like this, it's going to curve like this. It's going to come down and it's going to dip down below minus six to the left of the vertical axis, and then here, it's going to join up like this. Then it's going to keep going up like this.

00:05:19:46 - 00:06:17:91
Narrator: There's a graph of this quadratic function and the roots of the equation are where the parabola crosses the X axis, the horizontal axis and it crosses at x equal two and x equal minus three. Let's look at another example, few examples to see whether this always happens, whether we always have a quadratic equation like this, have two solutions because the graph is crossing in two places, crossing the horizontal axis in two places. We're actually going to see that that's not always the case. Let's do an example of minus X squared minus two X equals zero. So I'm going to factor this.

00:06:17:91 - 00:06:54:71
Narrator: I'll have minus X times X plus two equals zero. The two roots for this equation are going to be X equals minus two or zero. So we've ended up with two solutions for this example as well. In this case, the coefficient on the X squared term is negative. So rather than an upward opening parabola, this will be opening downwards.

00:06:54:71 - 00:07:37:00
Narrator: So it's going to be more like a hill than a valley. It's going to pass through the points X equal zero and X equals minus two. Again, if this is the X axis and this is the Y axis, and we've got zero, we've got minus one minus two. It's going to cross here and here and it's going to look more like this shape. It's it's going to be like this.

00:07:40:11 - 00:08:20:67
Narrator: That's this first example on this side. Second example on this side, let's do four x squared -12 X plus nine. Equals zero. Let's see if I can factor this thing. If I have two X and then if I have minus three, is that going to work?

00:08:20:67 - 00:08:47:34
Narrator: Let's see. Two X times two X would be four X squared and then minus three times minus three would be plus nine. Then how many Xs do I have? Minus three times two, I've got minus six x and then another minus six x -12 X, so I did it. So the other way to write this, of course, is just just write this.

00:08:49:86 - 00:09:25:70
Narrator: Here there's actually only one root for this equation. It's going to be when two X minus three is zero, which is when X equals 3/2. What does this graph look like if it only crosses or touches the horizontal axis in one place. It only touches at X equals 3/2, which is going to be here. Than this one.

00:09:28:06 - 00:10:04:93
Narrator: The coefficient on X squared is positive, so this is going to be more like a valley and it's going to look like this actually. That's why it only touches in one place because the bottom of the parabola actually touches the horizontal axis at 3/2. One root or two roots, is it possible that we could have zero roots? Let's do a third example. Let's do.

00:10:04:93 - 00:10:32:23
Narrator: Well, let's go back here and this was example number one. This is example number two, and this is example number three. Let's do a fourth example. Let's do X squared plus four x plus five equals zero. Okay, it doesn't matter how long I stare at this.

00:10:32:23 - 00:11:08:88
Narrator: I'm not going to be able to factor this. It doesn't factor. But there's a function close to this one that does factor, which would be X squared plus four x plus four. If I write this, then I can see that this is X plus to squared because I've got X squared, I got four X and I got four plus one. Equals zero. I need to try and make this equal to zero.

00:11:08:88 - 00:11:43:96
Narrator: Find a value of X so that this is equal to zero, can't do it because X plus two squared is the smallest this could be a zero. If X was minus two, this would be zero. But then I've still got the one here and so I can't get the left hand side of this equation equal to zero. This quadratic equation has no roots. What does it look like in terms of a graph?

00:11:45:28 - 00:12:33:42
Narrator: It turns out that this one is when X is minus two, then the function is equal to one. It's got a positive coefficient on the X squared, so it opens up upwards. It's more like a valley and it passes through this point here minus two and one. And it looks like this. It doesn't ever cross the horizontal axis.

00:12:35:14 - 00:13:30:93
Narrator: Quadratic equations like this, in general, like this, they either have two roots or one root or zero roots. If you're trying to visualize what that looks like, you could think about these three graphs. So this is a good point at which to pause the video and work through the next two problems that I'll give you and see if you can find the roots of the following two quadratic equations. The first one is going to be minus X squared plus four equals zero, and then the second one will be. That one's fairly easy.

00:13:30:93 - 00:14:11:45
Narrator: That shouldn't take you very long to do. This next one is a little bit more challenging because I'm going to throw some fractions in here. 3/2 X squared plus one quarter X, -15/4 equals zero. Just a hint for this one, pull out a fraction of one quarter. First, otherwise, you're going to have a heck of a time.

00:14:12:13 - 00:14:53:47
Narrator: Let's see how you did. First thing, I'm not super keen on this negative sign in front of the X squared. What I'm going to do is first thing I'm going to do is I'm going to multiply both sides of this equation by minus one just so that I don't have a negative on the X squared because that's awkward. If I multiply everything on this side by minus one, then it'll become X squared minus four, and then minus one times zero is zero. Then now I recognize this as a difference of squares.

00:14:53:47 - 00:15:18:94
Narrator: This is going to be X minus two times X plus two. Equals zero. X is either minus two or plus two. And once you've got your solution, remember always check it. So let's plug in minus two into my original equation.

00:15:18:94 - 00:15:40:74
Narrator: Minus two squared would be four minus four plus four is zero. That's good. Then two squared is four minus four plus four is zero. They both work out. What about this second one?

00:15:40:78 - 00:16:42:62
Narrator: I suggested that we pull out a factor of one quarter before we start just to make it a little bit easier to work with rather than dealing with these fractions. If I'm pulling out a quarter in front of the 3/2, then I need to have something a fraction here so that when I multiply it by a quarter, I get 3/2. If I have six, then 6/4 is the same as 3/2. Then for the X, that one's easy, a quarter times X, and then the 15/4, that one's easy too. See if I can factor this thing.

00:16:42:62 - 00:17:15:69
Narrator: So I want to get six X squared. One way I can get six X squared is if I have a three X and a two X. Let's start there. That would give me my six x squared. Then I'm going to need a -15 from my constants.

00:17:16:56 - 00:17:50:44
Narrator: So two factors of 15 could be three and two, three and five, not three and two, three times five is 15 and one will have to be positive and one will have to be negative. Then I need to get just plus one X. Let me see. We're going to have a five and a three. If I have five here, then I'm going to have ten X.

00:17:51:59 - 00:18:13:75
Narrator: And that would mean the three would be here and I'd have nine X. The difference between ten x and nine x is one X. I'll put three there. Sorry, I said I was going to put the five there. And the three here, and then which way round do I have to put the plus and the minus?

00:18:13:75 - 00:18:52:67
Narrator: I need to have a plus one X, so I need to have plus ten X from the five times two and minus nine X from minus three times three, and then plus five times minus three is -15. I think that factors it. That implies from the zero factor theorem that either this is zero or this is zero. If this is zero, then X must be equal to -5/3. If this is zero, X must be equal to 3/2.

00:18:53:23 - 00:19:16:91
Narrator: I've got two roots. One is -5/3 and one is 3/2. Let's just check. Let's check -5/3 first. Remember to check, I need to go back to my original equation.

00:19:16:91 - 00:20:09:42
Narrator: I'm going to have 3/2 times -5/3 squared, -5/3 squared would be 25/9. Then I've got a quarter times X, so one quarter times -5/3. Then finally, I've got -15/4 and that is going to have to come evaluate to the number zero. So a little bit of stuff going on here. The three and the nine.

00:20:09:42 - 00:20:51:49
Narrator: I could do a little bit of canceling here to be left with three in the denominator. I'm going to have 25/6. Then I've got -5/12. Then I've got -15/4. And I need to get a common denominator here to be able to add these up.

00:20:51:49 - 00:21:13:29
Narrator: A common denominator is 12. 25/6 is the same as 50/12. Then -5/12. This is, it is. It's going to work.

00:21:13:29 - 00:21:51:43
Narrator: Um, I've got -5/12 and then to get 15/4 as a fraction over 12, I have to multiply by three, it'll be -45. Now I've got 50 minus five -45, so I've got zero. That one checks out, we check this. Then let's check the 3/2. I'll do that here, 3/2.

00:21:52:27 - 00:22:58:35
Narrator: Times 3/2 squared be 9/4 and then plus one quarter times 3/2 and then -15/4. That's going to be 27/8 plus 3/8 -15/4. If I want to express that as a fraction over eight, it

will be -30/8. Equals zero, that one checks out as well. That's it for this video and in the next video, we'll stay with quadratic equations, but we'll figure out a way of solving them using something called the quadratic formula, which is something that we can use all the time.

00:22:58:35 - 00:23:04:65
Narrator: But it's particularly helpful when we can't factor in the way that we've been doing.