

16: Solving Linear Equations

Solving a linear equation in one variable

Define a linear equation in one variable as $ax + b = 0$, where a and b are real numbers and $a \neq 0$. To solve this equation means to find all values of x that satisfy this equation, i.e., the left-hand and right-hand sides of the equation are equal. This is also known as finding the *root* of the equation.

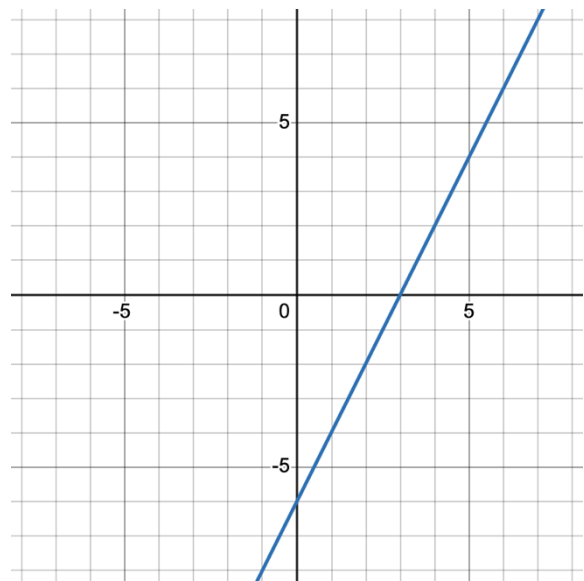
To solve equations like this we can add the same number to both sides of the equality or multiply both sides of the equality by the same number. The idea is to isolate x on one side of the equation with just numbers on the other side.

For example, $2x - 6 = 0 \Rightarrow 2x - 6 + 6 = 0 + 6 \Rightarrow 2x = 6 \Rightarrow \left(\frac{1}{2}\right) 2x = \left(\frac{1}{2}\right) 6 \Rightarrow x = 3$.

I strongly recommend checking the solution after solving an equation to make sure the solution works and to catch any errors. In this case, $2(3) - 6 = 6 - 6 = 0$, so the solution $x = 3$ works.

Other types of equation can have no solutions or more than one solution, but this type of equation only has one solution. This is easy to see on the graph of the function $y = 2x - 6$ to the right.

The graph has a y-intercept at -6 and a slope (rise/run) of 2. The solution is when $y = 0$, where the graph crosses the x-axis.



More examples

Find the value of x so that $3x - 4 = 8$.

- $3x - 4 = 8 \Rightarrow 3x = 12 \Rightarrow x = \frac{12}{3} = 4$.
- Written in the form $ax + b = 0$: $3x - 12 = 0$.
- Check: $3(4) - 4 = 12 - 4 = 8$.

Find the value of x so that $-\frac{5}{4}x + 2 = -3$.

- $-\frac{5}{4}x + 2 = -3 \Rightarrow -\frac{5}{4}x = -5 \Rightarrow x = \frac{4}{5}(5) = 4$.
- Written in the form $ax + b = 0$: $-\frac{5}{4}x + 5 = 0$.
- Check: $-\frac{5}{4}(4) + 2 = -5 + 2 = -3$.

Find the value of x so that $-2x + 3 = 1$

- $-2x + 3 = 1 \Rightarrow -2x = -2 \Rightarrow x = \frac{-2}{-2} = 1$.
- Written in the form $ax + b = 0$: $-2x + 2 = 0$.
- Check: $-2(1) + 3 = -2 + 3 = 1$.

Solving two linear equations in one variable

Another way that solving a linear equation can come up is finding the value of x that makes two linear equations in x equal. For example, suppose one equation is $y_1 = 2x - 3$ and the other equation is $y_2 = -3x + 7$. Set them equal and solve:

$$2x - 3 = -3x + 7 \Rightarrow 5x = 10 \Rightarrow x = 2.$$

Check: $2(2) - 3 = 4 - 3 = 1$ and $-3(2) + 7 = -6 + 7 = 1$.

Another example

- Find the value of x so that $\frac{5}{3}x - 9 = -2x + 2$.
- $\frac{5}{3}x - 9 = -2x + 2 \Rightarrow \frac{11}{3}x = 11 \Rightarrow x = \frac{3}{11}(11) = 3$.
- Check: $\frac{5}{3}(3) - 9 = 5 - 9 = -4$ and $-2(3) + 2 = -6 + 2 = -4$.

