



Understanding Integer Exponents

Transcript

Instructor: Iain Pardoe

00:00:00:00 - 00:00:53:03

Instructor: Hello, and welcome to video number six in this series. In this video, I'll go over the basics of numbers with integer exponents. The number A with an exponent N means A times A times A N times. Here, A is called the base, and that's any real number, and then N is the exponent and that's an integer exponent. For example, we could have two to the power five.

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Instructor: That's going to be two times two times two. We're going to multiply 2 five times. If we evaluate that, that comes to 32. A few more examples. Remember, the base can be any number, so it could be a fraction.

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Instructor: A half raised to the power five is a half times a half times a half, five times. That's going to be $1/32$. We could have a negative number as the base, minus two to the power five. Here we'll have minus two times minus two, five times. The twos, that will evaluate to 32 again.

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Instructor: Then to figure out whether it's plus or minus, we have to figure out how many minuses we have. If we have an even number of minuses, then we would have a positive result. But here we have an odd number of minuses, we're going to have -32. The base can be any number, so it could even be an irrational number like π . π raised to the power five.

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Instructor: It's just π times π times π times π times π . I don't even know what that is. It's a decimal, but I'm not interested in that at this point. I'm just interested in knowing what that notation means. π raised to the power five just means π multiplied by itself five times.

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Instructor: Let's go through a couple of examples. If I have A with an exponent of one, that's just A by itself. For example, two to the power one is just the number two. Anything raised to the power one is just itself. A raised to the power zero is defined as the number one.

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Instructor: For example, to the zero is just the number one. This works for any real number for the base A . It can be a little bit tricky thinking about zero to the power zero, but at least for some branches of mathematics, zero raised to the power zero is defined to be one. We generally don't worry about that kind of thing, and we're normally working with bases other than zero, and so any base raised to the power zero is one. Now let's think about negative numbers as the exponents.

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Instructor: For example, let's start with a to the minus one. So this is defined as the reciprocal of A . So for an example, two to the power minus one is $1/2$. If I have any negative integer as the exponent, then that is defined as the reciprocal of A to the N . For example, two to the minus five is $1/2$ to the five and two to the five, we calculate it up here is 32, so that's $1/32$.

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Instructor: But here's a couple more examples with negative exponents. I'm going to start with one over A , and then I'm going to have an exponent of minus one. Remember what we did over here when I had A with an exponent of minus one, it was the reciprocal of A . This time I'm starting with the reciprocal of A and I've got an exponent of minus one. If I take the reciprocal of a reciprocal, I end up back with what I started with.

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Instructor: This is actually equal to A . So for example, a half to the power minus one is two. Then just to push this one more step, let's start with one over A and then raise it to the power of minus N . We have to look at what we did here and combine it with this idea of the reciprocal of a reciprocal is the thing itself. So we'll have A to the N .

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Instructor: An example is a half to the minus five equals the reciprocal of a half is two, and we have two to the positive of the minus N . Two to the five. A couple more examples. Let's do minus two to the minus five. That's going to be one over minus two to the five minus two to the five was -32.

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Instructor: We're going to have one over -32, which is $-1/32$. Again, the base can be any real number, it can be an irrational number like π , So π to the minus five. That's going to be one over π to the five. You can also write this as one over π to the five like that. Let's explore a few more complicated expressions involving exponents.

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Instructor: Let's start with a B and then an A to the N. We're multiplying B by A to the N. We just have to be a little bit careful with the order of operations here. As an example, let's do B is three and A is two and let's have NB five. This is equal to three times two to the five and two to the five, that is an example we did earlier, that's 32, three times 32, which is 96.

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Instructor: You got to be a little bit careful here that you're not going to try and do the three times two first and then raise it to the power of five because that's a different thing entirely. Because that's going to be 6 raised to the power five, six times six times six, five times, it's going to be a lot bigger than 96. What about B times A to the minus N. So remember A to the minus N is over here. That's one over A to the N. We could rewrite this

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Instructor: as B over A to the N. An example here would be if B is three and A is two and N is five, we're going to have $3/32$. One last example, let's do B divided by A to the minus N. We can think of this as B times one over A to the minus N because I've got A to the minus N in the denominator. The reason I've written it like this is that I can match it up with something we did earlier.

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Instructor: We've got one over A to the minus N is the same as A to the N. This is actually the same as B times A to the N. These two expressions are equivalent. Pause the video at this point and see if you can figure out what the next two expressions look like if you evaluate them and simplify them as far as possible. For the first one, we're going to have four times minus two and the minus two is raised to the power of minus three.

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Instructor: For the second one, let's do minus three divided by minus four to the power minus two. There's a lot of things going on here, you're going to have to put together things that we reviewed earlier to figure out what these two expressions evaluate to. So for the first one, we've got a negative exponent in the numerator. We don't have a denominator here. There's no fraction here.

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Instructor: There's nothing in the denominator. It's all in the numerator. We've got minus three in the numerator. Let's go back over here and see what we have to do when we have a negative exponent in the numerator. We're in this situation here.

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Instructor: We've got something with a negative exponent, so we can rewrite that as one divided by the thing, A, whatever that is, in this case, it's going to be minus two with now a positive exponent, but we have to put it in the denominator to do that. Let's do that. We'll leave the four in the numerator and now in the denominator because now I'm turning this into a fraction, I'm going to have minus two, the minus two is raised to the power of the

positive of this what is currently a negative integer. Three. That's going to be four divided by minus two raised to the power three.

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Instructor: Minus two ways to the power three is just minus two times minus two times minus two, which is minus eight. We're going to have four over minus eight, and that is not in lowest terms. We've got to reduce this to lowest terms. There's a common factor of four in numerator and denominator. This is going to be equal to minus one half.

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Instructor: For this one, I'm starting off with a fraction where in the denominator, I've got something with a negative exponent. We're in this situation. I've got something in the denominator with a negative exponent. If that's the case, I can rewrite this by putting the thing that's in the denominator in the numerator with the positive of that negative exponent. So I'll keep the minus three and then I'll move the minus four to the numerator and then the negative exponent now becomes a positive exponent.

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Instructor: We're going to have minus three times minus four squared, minus four squared is positive 16 because it's minus four times minus four. The two minuses balance each other out. We get positive 16, positive 16 times minus three will now be -48. These two examples, everything that we've done up to this point in this video series, this is starting to get a little bit hairy now. But if you just go through each step step by step, applying what we've learned, you should be able to make sense of these two examples.

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Instructor: If you tried to do these and you couldn't quite get there, maybe review the video again and review what we did here. What I'm looking to do here, kind big picture is anytime I see a negative exponent, those are difficult to deal with and ideally what you want to do is you want to get rid of those negative exponents, turn them into positive ones by either moving something that's in the numerator like this into the denominator and switching the minus three to a plus three or vice versa, moving something that's in the denominator with a negative, moving it into the numerator with a positive. That's how to deal with negative exponents. You switch them, move them either up or down, and change the negative exponent into a positive one, and then you can do the evaluation. That's it for this video.

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Instructor: In the next video, we're going to continue working with numbers with exponents, but we're going to start to use arithmetic with them.