TRU
Open Press

Understanding Rational Exponents

Transcript

Instructor: Iain Pardoe

00:00:00:00 - 00:01:08:76
Narrator: Hello, and welcome to video number eight in this series. In this video, I'm going to continue our exploration of exponents by looking at rational exponents and roots. If we raise a real number A to the power one over N where N is an integer greater than one, then we obtain the principal nth root of A, which we write with a radical sign and a small N here. If A is positive, then the principal nth root of A is the real number B such that B raised to the power N is equal to A. For example, the principal third root of eight is two because to the power three is eight.

00:01:08:76 - 00:01:59:86
Narrator: Similarly, the principal second root of 16, and this is just called the square root. We often just omit two when we're dealing with square roots. The square root of 16, the principal square root of 16 is 4 because 4 squared equals 16. Now, we also know that minus four is a square root of 16 because minus four squared is 16, but minus four is not the principal square root, plus four is the principal square root. That's the case when A is positive.

00:02:00:06 - 00:02:47:47
Narrator: When A is negative, it depends whether N is even or odd, whether we even have a solution. I N is odd, then we're okay. We do have a solution and B is the real number, this time a negative real number such that B to the N is equal to A. To give you an example of A being negative, let's take the principal third root of minus eight. That's minus two because minus two cube is minus eight.

00:02:51:14 - 00:03:56:72
Narrator: If A is negative though and N is even then the principal nth root of A is not a real number. An example here would be, I've tried to take the square root of minus four, then there is no such real number that I could square to get minus four. The only other thing that we need to worry about here is I haven't really talked about when A is equal to zero. But

that's easy because the principle nth root of zero is just zero. Let's just go through a few more examples.

00:03:56:88 - 00:04:44:83
Narrator: Let's do the principal third root of 27. That's three because three cube is equal to 27. Let's do the principal fifth root of -32. We can do that because five is an odd number and we've got a odd base here, and that comes to minus two because minus two to the power five is equal to -32. Then let's throw a fraction in here.

00:04:44:83 - 00:05:40:72
Narrator: Let's do the principal fourth root of 1/81. That's going to be 1/3 because 1/3 to the power four is the same as 1/3 to the four, which is 1/81. Here, minus one third is a fourth root of 1/81, but it's not the principal fourth root. Let's expand this idea with exponents of the form one over N. Let's consider what it means to have an exponent that looks like this.

00:05:41:24 - 00:06:14:29
Narrator: Over N. What does that mean? We could either think of this as the principle nth root of A to the power M. That's one way to think about it the other way is to think about the th principal root of A to the M. So both ways are a legitimate ways to think about this.

00:06:14:29 - 00:06:35:24
Narrator: But in practice, it's generally easier to work with this form than this form. So I'll just run through a few examples to illustrate that. There's still a easy example first. Let's do eight to the 2/3. Think about this form first.

00:06:35:24 - 00:07:40:77
Narrator: We're going to have the third principal root of eight, and then we're going to raise that to the power two. The third principle root or the principle third root of eight is two and two squared is four. If we think about it the other way around, we would have the principal third root of eight squared, eight squared is 64 and the third root of 64 is four. So the arithmetic involved here is a little bit more complicated than here because here we had to take the principal third root of 64 and I know that that's four because I know that four cubed is 64, but there wasn't anything difficult to do here. The principal third root of eight is two and then two squared is four.

00:07:40:77 - 00:08:14:22
Narrator: This is easy and this is a little more challenging. Let's do an example with a negative number in. Let's do minus eight to the two thirds. Let's do the first method because that's a little easier to work with. That's going to be the principal third root of minus eight, and then we're going to square that.

00:08:14:22 - 00:08:32:20
Narrator: That's actually also going to be four because the principal third root of minus eight is minus two. That was an example that we did over here. But minus two squared is just plus four. We end up actually with the same answer here. Let's do another example.

00:08:32:20 - 00:09:06:08
Narrator: Let's do 81. Let's do a negative fractional exponent. 81 to the minus three quarters. Let's just write down what that means first. It's going to be the Well, actually, because I've got a negative exponent here, I'm going to the first step I want to do is I just want to push the 81 into the denominator and make this positive.

00:09:11:76 - 00:09:42:49
Narrator: It's the same as 1/81 to the three quarters exponent. Then I can write down what this means. It's the principal fourth root of 81 cubed. The fourth root of 81 is three and three cubed is 27. This comes to 1/27.

00:09:43:17 - 00:10:24:03
Narrator: Then let's do one last example here. Let's do nine 16th to the 3/2 exponent. Let's write down what that means. That's going to be the square root of 9/16 cubes. Square root of nine 16th is 3/4 and three cubed is 27 and four cubed is 64, so that's going to be 27/64.

00:10:28:11 - 00:10:59:34
Narrator: That's working with numbers. We can apply the same ideas when we work with variables as well. So let's do a couple of examples with variables. Let's do nine X squared Y to the four. Then let's have a 3/2 power there.

00:10:59:54 - 00:11:17:06
Narrator: Let's think about what that is. Let's do the constant first. We got nine to the 3/2. That's the square root of nine cubed. Square root of nine is three, three cubed is 27.

00:11:17:42 - 00:12:01:23
Narrator: Got X squared to the 3/2. When I've got an exponent, then I've got another exponent, I multiply them together, two times 3/2, just B three. I've got X cubed, and then y to the four to the 3/2, multiply four by 3/2 and I'm going to have six. Let's do one more example. Let's do 1/64 and then X cubed, and then Y to the minus six.

00:12:01:23 - 00:12:28:44
Narrator: Then let's have that B raised to the power of one third. Deal with the constant first. It's got 1/64 to the one third power. That's the principal third root of 64, which is four and it's in the denominator. About four in the denominator.

00:12:28:44 - 00:13:04:94
Narrator: Now I've got X cubed to the one third. Three times one third is just one. I'll have X in the numerator and then I got Y to the minus six to the one third, minus six times one third is minus two. Y to the minus two is the same as having Y squared in the denominator. Pause the video here and see if you can work through the next two problems that I'll write up.

00:13:04:94 - 00:13:38:60
Narrator: And what you want to do is apply the ideas that we've been talking about here, particularly these last two because the next two examples will be more like these two. I

want you to work through and simplify these two expressions. For the first one, let's do 16. Got to do that again. We have a different pen.

00:13:40:92 - 00:13:55:80
Narrator: The colors. I know. It's uh Yeah. It's a shame. I like the orange. Yeah. Well, maybe that one will work.

00:13:55:80 - 00:14:53:16
Narrator: Yeah, give it a go anyway. For this first one, we're going to have 16 and then X to the half, and then Y to the minus two, and then three quarters. Then for the second one, let's do X to the minus four and then Y squared, and we'll have minus a half. Actually, I'm going to add a little bit more to this. Let's divide this by 49 before we do the minus a half.

00:14:55:06 - 00:15:15:82
Narrator: Let's see how you did with these two examples. Let's deal with constant first in the first one. We've got 16 to the three quarters. That's going to be the principal fourth root of 16, which is two cubed. It's going to have eight.

00:15:15:82 - 00:15:42:32
Narrator: Then we've got X to the half to the three quarters. That's going to be X to the three eighths. Then we've got Y to the minus two to the three quarters. That's going to be Y to the -3/2, which is the same as having X to the plus 3/2 in the denominator. Sorry, Y to the 3/2 in the denominator.

00:15:46:00 - 00:16:20:00
Narrator: Then the second one, let's deal with the 49 in the denominator first. We got 49 with a negative one half exponent. One half exponent means the square root, the square root of 49 is seven. But the negative means I have to move that from the denominator into the numerator. We'll actually have seven in the numerator.

00:16:20:00 - 00:17:11:54
Narrator: And then we got X to the minus four to the minus a half, minus four times minus a half is plus two. That means X squared in the numerator, and then we got Y squared to the minus half, two times minus a half is minus one, we got Y in the denominator. So before we finish up this video, I'm just going to explore a notion called rationalizing the denominator because sometimes, not always, but sometimes we're trying to end up with a answer at the end that doesn't have any roots in the denominator. I'm going to show you how to remove roots from a denominator if that's your desired outcome. We'll start with just a very simple example, one over root two.

00:17:11:54 - 00:17:56:95
Narrator: If I want to rationalize the denominator here, I need to multiply this root two by something so that we can remove that root. The number that works here is just multiply by another root two because then root two times root two will be two and we've removed the root from the denominator. To do that, we need to be multiplying by a fraction that's

equivalent to one and so we just do root two over root two. Now if we multiply the numerators, we get root two. If we multiply the denominators, we get two.

00:17:57:34 - 00:18:39:32
Narrator: For another example, to one that's similar, but just a little bit more challenging. Let's do the principal third root of two in the denominator. What do I need to multiply by to get rid of this? Well, if I multiply by the principal third root of two squared, then if I multiply these denominators, I'll be able to get rid of the root. I got to have in the numerator the same thing.

00:18:41:27 - 00:19:33:13
Narrator: Now when I multiply the numerators, I'll have the principal third root, I'll just write four instead of two squared and then in the denominator, I've got the principal third root of two and then the principal third root of two squared. Multiplying these together, I've got the principal third root of two cubed and the principal third root of two cubed is just two. So that's some examples just with numbers. Let's apply the same idea with expressions that contain variables. Let's suppose I have this expression here, X over 2Y and the one third power.

00:19:34:13 - 00:20:25:00
Narrator: Which is fine as it is, but we do have a fractional exponent in the denominator here. We have a root in the denominator. If you want to rationalize the denominator here, what we need to do is multiply by Y, if we've got the one third power to get that up to a power of one, we would need to have two Y to the two thirds power. Because now these two multiplied together would just give us two Y. We've got two Y to the two thirds.

00:20:25:84 - 00:20:41:58
Narrator: So it's easy enough to see what we've got in the denominator now. We've just got two Y. But what do we have in the numerator? We've got X to the one third. We've got two to the two thirds and we've got Y to the two thirds.

00:20:41:58 - 00:21:31:59
Narrator: Let's think about what we would need to write here to have everything to the one third. We've got two squared to the one third, four to the one third power. We've got X to the one third, and we've got Y squared to the one third. So writing this using the radical notation, we'll have the principal third root of four X Y squared over two Y. That's it for this video.

00:21:31:59 - 00:21:37:57
Narrator: In the next video, we're going to switch gears a little bit and start looking at polynomials.