Understanding Real Numbers and Basic Arithmetic Properties

Transcript

Instructor: Iain Pardoe

00:00:00:43 - 00:00:41:75
Instructor: Hello, and welcome to this first video in this series. In this video, I'm going to go over the real number system and review some basic arithmetic properties. Real numbers are real in the sense of representing physical measurements of things in the real world. There's actually a whole collection of numbers called imaginary numbers used in some branches of mathematics, but we're going to stick with real numbers for this video series. Within the real numbers, there's rational numbers.

00:00:42:63 - 00:01:39:42
Instructor: Which represent fractions and quotients, then there's irrational numbers, which are any real numbers that are not rational, so those are numbers like Pi square root of two and that kind of number. Rational numbers can be written as either terminating or repeating decimals. An example of a terminating decimal would be two fifths, which as a decimal is 0.4, so that decimal terminates, it stops. An example of a repeating decimal would be one third.

00:01:39:42 - 00:02:19:14
Instructor: If it goes as a decimal number, that's 0.3, three, and the three just keep going so they keep repeating. By contrast, irrational numbers can only be represented as non terminating, non repeating decimals. For example, Pi as a decimal is 3.14, one, five, nine, two, and the numbers keep going with no pattern, and root two, is 1.41, four, two,

00:02:19:14 - 00:02:55:97
Instructor: one, three, and they keep repeating two with no pattern. Within the rational numbers, there are the integers. And we have negative integers, positive integers and zero and the positive integers are also called natural numbers. If we include zero, then these are called the whole numbers. Anytime we're working with numbers, it can be useful to specify which subset of the real numbers we're working with.

00:02:55:97 - 00:03:29:47
Instructor: If we're working just with whole numbers with integers, including negative numbers, maybe we're working with rational numbers, maybe we're also including irrational numbers to the entire real number set. Next, let's run through the basic arithmetic properties. that will need to become second nature to you the further you go in your mathematical journey. We'll start with addition. So if I add two numbers together, it doesn't matter which order I add the numbers.

00:03:29:47 - 00:03:54:01
Instructor: A plus B is the same as B plus A. Very example, three plus four is the same as four plus three. It's seven, whichever order I add the two numbers together. I have three numbers. A plus B plus C. If I add the A and the B first and then add C,

00:03:54:01 - 00:04:33:28
Instructor: I'll get the same result as if I add the B and the C first and then add the A. For example, three plus four plus two. If I add the three and the four first, I'll get seven plus two. If I do it the other way around and add the four plus two and then add the three, I'll get three plus six and either way I'll get nine. If we have a number and we add zero to it, then the number doesn't change.

00:04:33:88 - 00:05:06:28
Instructor: If I have a number and I add the negative of that number, then I'll get zero. Next, multiplication. If I have two numbers that I multiply together, it doesn't matter which order I multiply them. A B is the same as BA. For example, three times four, it's the same as four times three, it's 12 either way.

00:05:06:32 - 00:05:58:33
Instructor: If I have three numbers and I want to multiply them together, I I multiply the first two numbers first and then multiply by the third number, I'll get the same as if I multiply the last two numbers first and then multiply by the first number. For example, three times four and then times two. That'll be 12 times two, 24, it's the same as if I do four times two first, which is eight and then three times eight is 24. Same answer either way. If I have a number and I multiply it by one, then the number will stay the same.

00:05:59:57 - 00:07:06:60
Instructor: If I have a number and I multiply by one over that number, and one over the number is called the reciprocal, then I'll get a number one. Next, if we multiply A times B plus C, then that's the same as A times B plus A times C. For example, I have three times four plus two, then one way I could calculate this is add the four and the two to get six and multiply by three. Three times six is 18. The other way I could calculate it is to use this result up here and I could do three times four.

00:07:06:89 - 00:07:35:74
Instructor: And add three times two. Three times four would be 12 and then three times two is six, and then 12 plus six is 18. I get the same answer either way. And this can be useful when we're solving more complicated equations. Sometimes it's easier to calculate.

00:07:35:74 - 00:08:42:30
Instructor: Using this method, sometimes it's easier to calculate using this second method. Building from this last property, if I now have A plus B times C plus D, we can expand this into four what are called partial products. The first partial product taken from the first two pieces, so the A and the C would be A, C, and the next two numbers that I'm going to multiply are the outer two numbers, A and D. Then the next two numbers I'll multiply, are going to be the inner numbers, B and C, and then the last two are B and D. So we had the first two and then the outer numbers, and then the inner numbers, and then the last two numbers.

00:08:42:30 - 00:09:36:17
Instructor: That spells out the acronym foil. This is typically known as foil, for example, if I have three plus four times two plus one, Just as in the previous example, I had two different ways of calculating. I'll do two different ways for this example as well. I could do this three plus four is seven and then two plus one is three and seven times three is 21, or I could use foil and I'll do the first two numbers. The three and the two. And then

00:09:36:53 - 00:10:14:88
Instructor: next one is the outer numbers three and the one. The next partial product is going to be the inner numbers, so the four and the two. Then the final one is the last numbers, the four and the one. Three times two is six, three times one is three, four times two is eight. Four times one is four, six and three is nine plus eight is 17 plus four is 21.

00:10:14:88 - 00:10:54:46
Instructor: I get the same answer either way. Again, just as in the previous example, sometimes in more complicated expressions, it's easier to work with this calculation method and sometimes it's easier to expand it and use this calculation method. Finally, I'm going to finish up thinking about equations and solving equations. I'm going to use this property here. I have an equation A equals B and I want to solve that equation.

00:10:54:46 - 00:11:43:56
Instructor: As long as I do the same thing to both sides of the equation, then that can be a way of simplifying the equation to solve it. For example, I can add the same number to both sides of the equation. A plus C is the same as B plus C, or it can multiply both sides of the equation by the same number. A times C is the same as B times C. For example, if I had the equation two X plus three equals nine, and I want to solve this equation for X. I want to isolate

00:11:43:56 - 00:12:19:72
Instructor: X on one side of the equation and just have numbers that I can evaluate on the other side of the equation. The first thing I want to do is I want to move that three from the left hand side of the equation and move it over to the other side. One way to do that is to subtract three from both sides of the equation. I'm using this idea here. So I'll have two X plus three minus three is equal to nine minus three.

00:12:19:84 - 00:13:12:39
Instructor: Now plus three minus three is zero, and I'm left with two X on the left hand side of the equation, and I'll have six on the other side. Next, we have to do something about the two here. I need to think of a number that I can multiply by two to get one so that I'll have one times X, which is equal to X. If we think back to one of the properties from earlier, I multiply two by the reciprocal of two, which is a half, then I'll end up with one times X and I'll have X on the left hand side, and then I have to do the same thing to the other side of the equation, multiply by a half. Now I've got one times two is one and one times X is just X.

00:13:12:39 - 00:13:54:10
Instructor: Then on the right hand side of the equation, I've got a half times six, which is three, and there's my answer. Anytime I solve an equation like this, it's always a good idea to do a check at the end that your answer does actually solve the equation that you are trying to solve. To do that, I'll plug in the answer that I got into my original equation. Two times three, and then we're adding three, and then we should be getting nine, two times three is six. And sure enough, six plus three equals nine.

00:13:54:34 - 00:14:11:50
Instructor: That's a good check that I've not made any mistakes during my series of manipulations here. That's all for this video. In the next video, I'll look at representing numbers as points along a number line.